Worksheet 2.6A, Rational functions
MATH 1410
(SOLUTIONS)

For each of the rational functions given below, do the following:

1. Find the domain of the rational function.
2. Reduce the rational function to lowest terms, if possible.
3. Find the x- and y-intercepts of the graph of the rational function, if they exist.
4. Determine the location of any vertical asymptotes or holes in the graph, if they exist.
5. Analyze the end behavior of the rational function. Find the horizontal or slant asymptote, if one exists.
6. Use a sign diagram and plot additional points, as needed, to sketch the graph of the rational function.

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1. $a(x) = \dfrac{2x^2 - 9}{x^2 - 9}$
2. $b(x) = \dfrac{x}{x - 1}$
3. $c(x) = \dfrac{x + 3}{x - 2}$
4. $d(x) = \dfrac{(x + 1)(2x - 2)}{(x - 3)(x + 4)}$
5. $e(x) = \dfrac{(2x - 1)(x + 2)}{(2x + 3)(3x - 4)}$
6. $f(x) = \dfrac{x^2 - 1}{x^2 + x - 6}$
7. $g(x) = \dfrac{x^2 - 4}{3x^2 + x - 4}$
8. $h(x) = \dfrac{x^2 - 6x + 8}{x^2 - x - 12}$
9. $i(x) = \dfrac{x^2 - 9}{x^3 - 4x}$
10. $j(x) = \dfrac{2x + 1}{x^2 + x + 1}$

Solutions.

1. $a(x) = \dfrac{2x^2 - 9}{x^2 - 9}$ has domain all real numbers except ± 3 so, in interval notation, the domain is

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty).$$

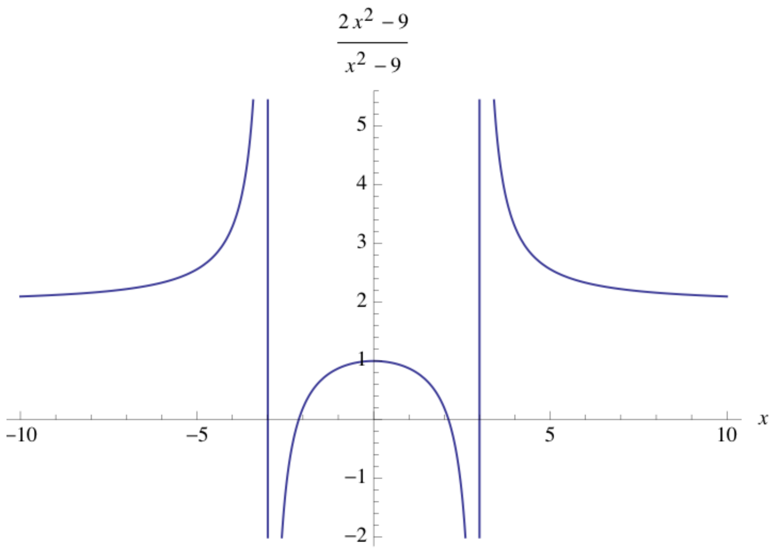
The rational function has zeroes when $x = \pm\dfrac{3}{\sqrt{2}}$.

Its y-intercept occurs when $y = 1$.

Vertical asymptotes are $x = 3$ and $x = -3$.

$y = 2$ is a horizontal asymptote.

Here is a graph of the curve, along with the two vertical asymptotes:



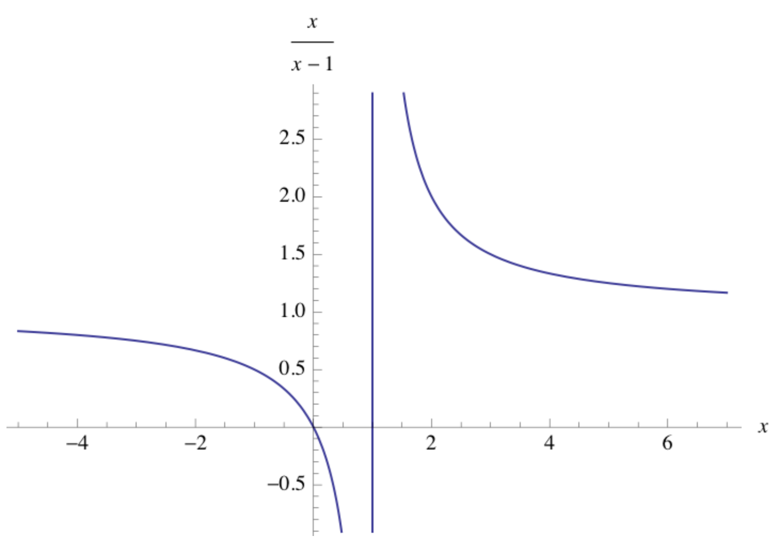
2. $b(x) = \dfrac{x}{x-1}$ has domain all real numbers except 1 so, in interval notation, the domain is

$$(-\infty, 1) \cup (1, \infty).$$

The rational function intersects the axes at the origin.

It has a vertical asymptote $x = 1$ and $y = 1$ is a horizontal asymptote.

Here is a graph of the curve, along with the one vertical asymptote:



3. $c(x) = \dfrac{x+3}{x-2}$ has domain all real numbers except 2 so, in interval notation, the domain is

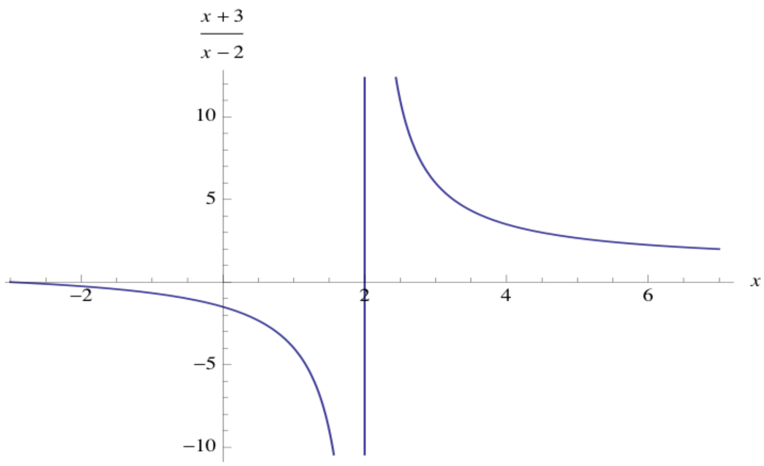
$$(-\infty, 2) \cup (2, \infty).$$

The rational function has zeroes when $x = -3$.

Its y-intercept occurs when $y = -\dfrac{3}{2}$.

Vertical asymptotes are $x = 2$; $y = 1$ is a horizontal asymptote.

Here is a graph of the curve, along with the one vertical asymptote:



4. $d(x) = \dfrac{(x+1)(2x-2)}{(x-3)(x+4)}$ has domain all real numbers except $-4, 3$ so, in interval notation, the domain is

$$(-\infty, -4) \cup (-4, 3) \cup (3, \infty).$$

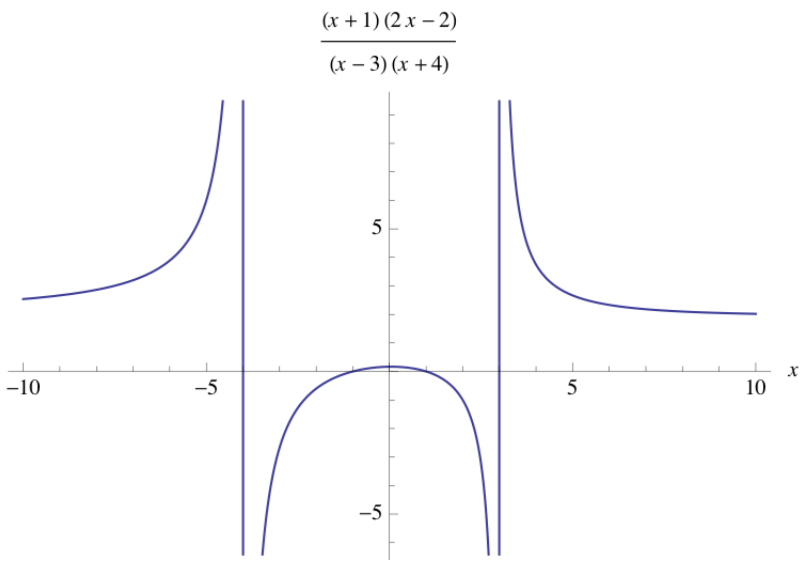
The rational function has zeroes when $x = -1$ or $x = 1$.

Its y-intercept occurs when $y = \dfrac{1}{6}$.

Vertical asymptotes are $x = 3$ and $x = -4$.

$y = 2$ is a horizontal asymptote.

Here is a graph of the curve, along with the two vertical asymptotes:



5. $e(x) = \dfrac{(2x-1)(x+2)}{(2x+3)(3x-4)}$ has domain all real numbers except $-\frac{3}{2}, \frac{4}{3}$ so, in interval notation, the domain is

$$(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \frac{4}{3}) \cup (\frac{4}{3}, \infty).$$

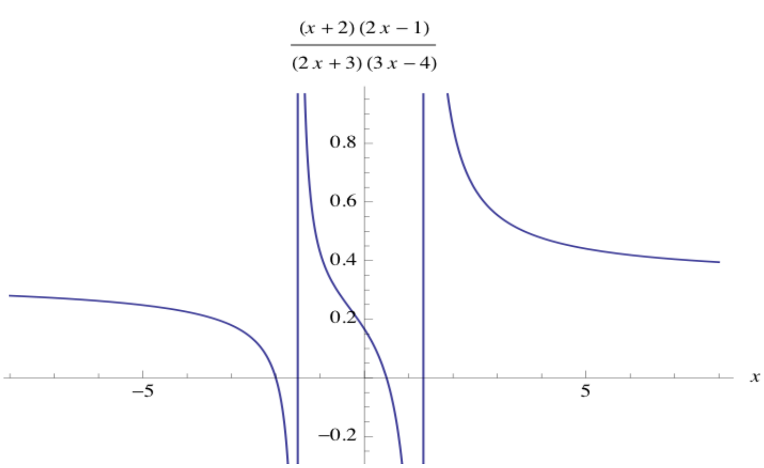
The rational function has zeroes when $x = \dfrac{1}{2}, x = -2$.

Its y-intercept occurs when $y = \dfrac{1}{3}$.

Vertical asymptotes are $x = -\dfrac{3}{2}$ and $x = \dfrac{4}{3}$.

$y = \dfrac{1}{3}$ is a horizontal asymptote.

Here is a graph of the curve, along with the two vertical asymptotes:



6. $f(x) = \dfrac{x^2-1}{x^2+x-6}$ has domain all real numbers except $-3, 2$ so, in interval notation, the domain is

$$(-\infty, -3) \cup (-3, 2) \cup (2, \infty).$$

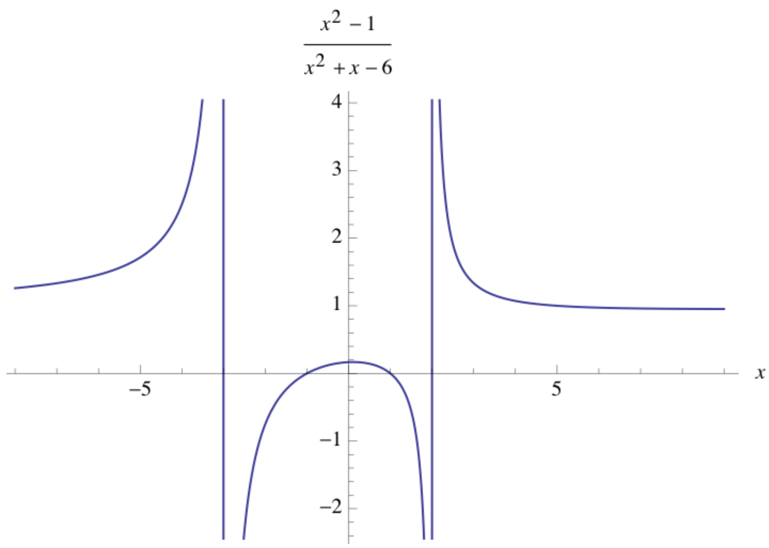
The rational function has zeroes when $x = \pm 1$.

Its y-intercept occurs when $y = \frac{1}{6}$.

Vertical asymptotes are $x = 2$ and $x = -3$.

$y = 1$ is a horizontal asymptote.

Here is a graph of the curve, along with the two vertical asymptotes:



7. $g(x) = \dfrac{x^2 - 4}{3x^2 + x - 4}$ has domain all real numbers except $-\frac{4}{3}, 1$ so, in interval notation, the domain is

$$(-\infty, -\frac{4}{3}) \cup (-\frac{4}{3}, 1) \cup (1, \infty).$$

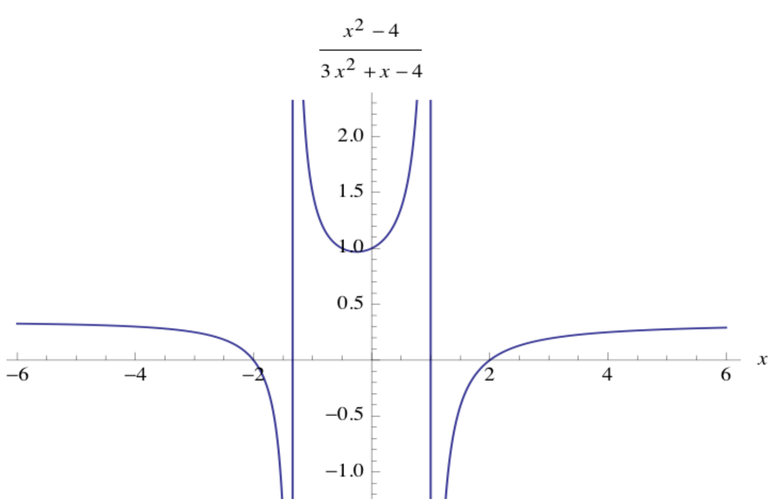
The rational function has zeroes when $x = \pm 2$.

Its y-intercept occurs when $y = 1$.

Vertical asymptotes are $x = 1$ and $x = -\frac{4}{3}$.

$y = \frac{1}{3}$ is a horizontal asymptote.

Here is a graph of the curve, along with the two vertical asymptotes:



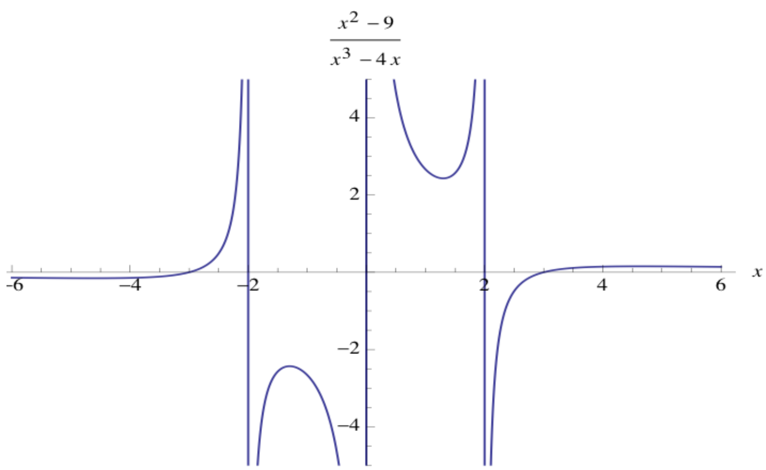
8. $h(x) = \dfrac{x^2 - 6x + 8}{x^2 - x - 12}$ (See the class notes.)

9. $i(x) = \dfrac{x^2 - 9}{x^3 - 4x}$ has domain all real numbers except $\pm 2, 0$ so, in interval notation, the domain is

$$(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty).$$

The rational function has zeroes when $x = \pm 3$. It has no y-intercept since $x = 0$ is a vertical asymptote. Other vertical asymptotes are $x = 2$ and $x = -2$.

$y = 0$ is a horizontal asymptote. Here is a graph of the curve, along with the three vertical asymptotes:



10. $j(x) = \dfrac{2x + 1}{x^2 + x + 1}$ has domain all real numbers since the denominator is never zero. (In interval notation the domain is $(-\infty, \infty)$.)

The rational function has zero $x = -\dfrac{1}{2}$

Its y-intercept occurs when $y = 1$.

It has no vertical asymptotes and $y = 0$ is a horizontal asymptote.

Here is a graph of the curve: