Lesson 3: Using Inequalities to Problem Solve

GEE 21
Graduation Exit Examination for the 21st Century

Selected Content Standards

Benchmarks Addressed:

N-1-M *Demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)*

N-2-M *Demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents*

A-1-H *Demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa*

A-2-H *Recognize the relationship between operations involving real numbers and operations involving algebraic expressions.*

A-4-H *Solving algebraic equations and inequalities using a variety of techniques with the appropriate tools (e.g., hand-held manipulatives, graphing calculator, symbolic manipulator, or pencil and paper)*

GLEs Addressed:

Grade 8

1. Compare rational numbers using symbols (i.e., $<$, $\leq$, $=$, $\geq$, $>$) and position on a number line (N-1-M) (N-2-M)

Grade 9

9. Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11. Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
14. Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)

Lesson Focus:

This lesson is intended to be an introduction to inequalities and an extension on solving equations. By the end of this lesson students should know the following:

- The meaning of inequality sentences involving $<$, $>$, $\leq$, and $\geq$.
- Write inequalities that describe a given situation
- Solve for the variable in multi-step inequalities

GEE 21 Connection

The skills that will be addressed in this lesson include the following:

- Solve and graph linear inequalities involving addition, subtraction, multiplication, or division.
- Solve and graph real-world problems (addition, subtraction, multiplication, division) involving linear inequalities.

Translating Content Standards into Instruction

A. Discuss with the students the differences between the inequality symbols $<$, $>$, $\leq$, and $\geq$. The teacher can begin by using the driver's license example in part A of the**Teacher Blackline #1**: *"A person must be at least 16 years old to get a Louisiana driver's license."* Ask the students what inequality could be used to represent this situation. The teacher should take the opportunity to discuss the difference between $>$, which does not include the minimum number, and $\geq$, which does include the minimum number. Also, discuss with the students what the possible replacement values for "*a*" would be. Could it be 21, 75, 36.7, 84 ½, zero, or a negative number? Emphasize that all of the ages greater than or equal to 16 could replace the variable for "*a*". Get the students to recognize that although all numbers greater than or equal to 16 are possible in the inequality, $a \geq 16$, in this particular situation (people with driver's licenses) there would be a limiting maximum age. Ask the students to graph the situation on a number line, and then discuss with the students why we graph the solutions and not list them.

   Discuss with the students how to tell the difference between words that indicate minus (less, less than) and words that indicate $<$ (is less than). Ask the students to come up with inequality statements for other examples on **Teacher Blackline #1**. While working through these examples in section A, discuss the connections between the words and the symbols. How do phrases like "is at least", "is at most", and "is greater than" translate into algebra? Discuss with the students how to graph each inequality. What numbers make sense as the answers?

B. Now we want the students to use their understanding of writing algebraic sentences from a problem situation to write more complex inequalities from problem situations.

   Have the students work in pairs to translate examples of situations in **Teacher Blackline #, Part B** that can be described by inequalities. The teacher should allow the students to see that in some cases there is more than one correct way to write the inequality. Discuss with the students whether alternative inequalities are valid for the particular problem.

C. As the students become more comfortable translating situations into algebraic inequalities, we want the students to explore to see if the Golden Rule for solving equations *("Whatever we do to one side of the equation, we must do to the other side in order for the two sides to remain equal.")* holds

true for inequalities. Get the students to write a true inequality involving only numbers (For example, 5 > 4.). Now ask the students to add the same positive number to both sides and determine if the inequality is still true. Then ask them to add the same negative number to both sides and determine if the inequality is still true. Ask the students to do the same (using a positive number and then a negative number) with the operations of subtraction, multiplication, and division. Discuss with the students that the Golden Rule does not hold true if both sides of the inequality are multiplied or divided by a negative number. Ask the students what could be done in those cases to make the inequalities true. The teacher should assist the students in formulating the property, "When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality sign to keep the inequality true."

The teacher should work through a few examples with the students in **Teacher Blackline #2, Part C** cautioning the students to be on the lookout for the situation when they will need to multiply or divide both sides by a negative. Require the students to graph the solutions to the first three problems.

Sources of Evidence about Student Learning

A. In the process of investigating inequalities, the students should be able to discuss what they are discovering about inequalities. The students should be able to translate phrases that indicate inequalities into algebra.

B. The students should be required to explain their thought processes to each other as they work through the examples. If the students are required to give explanations as to why and how they arrived at their answers, then they have to stop and think the problem through to organize their thoughts.

C. Have the students work through the **Student Worksheet**, discuss their answers in small groups, and revise their own work. Also, have students recognize the variety of correct ways that the problems were worked, by asking for examples and encouraging alternative methods of finding solutions.

D. Ask students to create a problem situation from a given inequality. Be sure to require the students to identify what the variable represents.

GEE 21 Connection

Sample items similar to what students might see on the GEE 21 test:

1) A restaurant has small tables, *s,* and large tables, *t*. Small tables seat four people each, and large tables seat eight people each. Which inequality shows the maximum number of people, *p,* that can be seated at the restaurant?

   a) $p \geq 8t + 4s$
   b) $p \leq 8t + 4s$
   c) $p > 8t + 4s$
   d) $p < 8t + 4s$

LEAP Released Test Items, Grade 8, July 2000

2) Ticket sales at the First Run Theater total at least $7,600 per week. An adult's ticket costs $7.50 and a child's ticket costs $4.00. If *a* represents the number of adult tickets sold in a week and *c* represents the number of child tickets, which algebraic sentence represents the money received each week from ticket sales?

   a) $7.50a + 4.00c = 7{,}600$
   b) $7.50a + 4.00c \geq 7{,}600$
   c) $7.50a + 4.00c > 7{,}600$
   d) $7.50a + 4.00c < 7{,}600$

Massachusetts Grade 10 MCAS Re-Test Study Questions, 2001

3) What is the least whole number, *x,* for which $2x > 11$?

   a) 5
   b) 6
   c) 9
   d) 22
   e) 23

NAEP

Attributes of Student Work at the "Got-It" Level

A. Students should be able to translate problem situations into appropriate inequalities, to solve those inequalities, and to use that solution to answer questions about a problem situation. When answering the questions in the problem situation, the students should be able to determine if their answers are reasonable.

B. Students should be able to make a connection between an inequality statement and its graph.

C. Students should be able to recognize and discuss alternative correct ways to work the problems.

A. "A person must be at least 16 years old to get a Louisiana driver's license."

What would the graph of that inequality look like? Why would we want to graph the solution? Why not just list the solutions?

Examples

1) The number of hours a full time employee can work a week without getting overtime pay is at most 40 hours.

2) In order to ride the Triple Threat Roller Coaster, a rider must be at least 42 inches tall.

3) The most number of cans collected by any student in the Save the Earth Club was 256. Write an inequality to express the number of cans ***c***, collected by any other club member.

B. Write as inequalities, but do not solve.

1) You and your friends have pooled your money and have $12 to spend on lunch. You agree to share a large fry and buy hamburgers with the rest of the money. If the price of a large fry is $1.22 and the price of a hamburger is $0.89, how many hamburgers can you buy?

2) Sam had $39 to buy Christmas for his family. He bought a $12 t-shirt for his sister, an $18 bracelet for his Mom, and still needs to get his Dad a CD. What is the most money he can spend on his Dad's present?

3) A long board must be split into four sections, so that each section is at most $3\frac{1}{2}$ feet long. What is the maximum length the board can be?

4) April started a workout program at a local health club. She began with a 45 minute workout and has increased her time by 5 minutes per week. How many weeks should she increase her time, if she wants her workout to be no more than one hour?

C. Solve the inequalities and graph on a number line.

1) $M - 4 < 9$ <———————————>

2) $6a \geq -24$ <———————————>

3) $5x - 7 > -22$ <———————————>

4) $3(x + 2) \leq 72$ <———————————>

5) $-5x + 14 \geq 34$ <———————————>

After working with the students through these three examples, the teacher should return to **Teacher Blackline #1**, Part B and ask the students to now solve the problems by solving the inequalities.

Translate the following into algebraic inequalities, solve the inequalities, and use that solution to answer the questions. Designate a variable for the unknown.

1) Mercury has the lowest melting point (-37.8°) of all metals. Write an inequality for the melting point of any other metal.

2) Dan wants to order some DVDs. An Internet company charges $19 per DVD plus $4.50 for shipping and handling on the entire order. If Dan wants to spend at most $75, how many DVD's can he buy? How much money will he have left over?

3) The perimeter of a square must be less than 160 feet. What is the maximum length of a side in feet?

4) Mrs. Jones needs to buy some Purrfect Brand Cat Food. If each can costs $0.79 and she has a coupon for $2 off the entire purchase, how many cans of cat food can she buy for under $7?

5) Admission to a carnival is $8.00. You allow $5.00 for lunch and $3.00 for a snack. Each ride is $2.50. You have $40.00 to spend. How many rides can you go on?

6) A cellular phone company advertises cut-rate phone service for $9.00 a month plus $0.45 per call. If your budget allows you to spend at most $15 on phone service a month, what is the maximum number of calls you can make?

Teacher Blackline #1

A. "A person must be at least 16 years old to get a Louisiana driver's license."

The graph is important because it is not possible to list all of the ages for which a person in Louisiana can get a driver's license. The person can be 16 or any age over 16. For example: 16 ½, 17, 25, 25.3, 99, etc. The graph would give us an accurate picture of the range of ages.

Examples

1) $h \leq 40$

2) $h \geq 42$

3) $c < 256$

B.

1) $1.22 + 0.89h \leq 12$ solution: ($x \leq 12.1123$, which means 12 hamburgers)

2) $12 + 18 + c \leq 39$ or $c \leq 39 - 12 - 18$ solution: ($c \leq 9$, \$9 is the most he can spend on his Dad's CD)

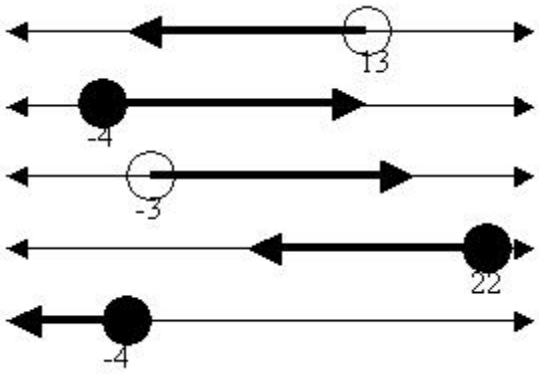
3) $\frac{x}{4} \leq 3.5$ solution: ($x \leq 14$, The maximum the board can be is 14 feet.)

4) $45 + 5x \leq 60$: solution ($x \leq 3$, April can increase her workout for only 3 weeks.)

Teacher Blackline #2

C.

1. $m < 13$
2. $a \geq -4$
3. $x > -3$
4. $x \leq 22$
5. $x \leq -4$



Student Worksheet

1) $m > -37.8$ degrees m = melting point

2) $19d + 4.50 \leq 75$ d = # of DVD's
$d \leq 3.71$
Dan can order at most 3 CD's. $75 - 3(19) - 4.50 = \$13.50$ left over

3) x = length of a side
$4x < 160$
$x < 40$
Each side must be less than 40 feet.

4) $0.79c - 2 < 7$ c = number of cans
$c < 11.392405$
Mrs. Jones can buy a maximum of 11 cans of cat food.

5) $8 + 5 + 3 + 2.50\,r \leq 40$ r = # of rides
$r \leq 9.6$
You can go on a maximum of 9 rides.

6) $9 + 0.45c \leq 15$ c = # of calls
$c \leq 13\frac{1}{3}$
You could make at most 13 calls on your cell phone per month.

GEE 21 Connection

1) a
2) b
3) b