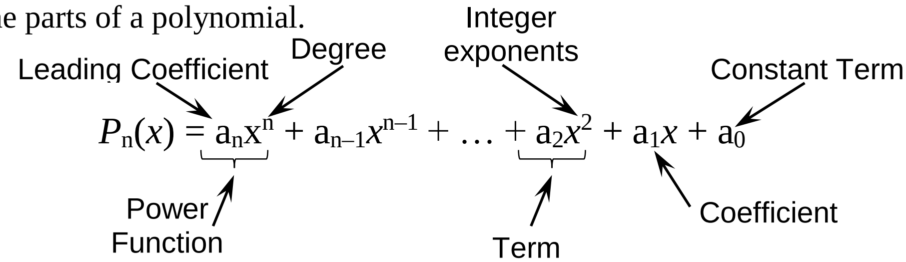
Operations with Polynomials

What is a Polynomial?

These are the parts of a polynomial.



$$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

Definition: A polynomial, $P(x)$, is a sum of multiples of nonnegative integer powers of a variable.

A polynomial in *standard form* is written in exponent order, highest to lowest power.
A polynomial function of degree n, $P_n(x)$, may be referred to by one of the following names:

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic
n	n^{th} degree

Terms	Name
1	Monomial
2	Binomial
3	Trinomial
>3	Polynomial

A **term** is a single product of constants and variables.
Two terms are **like** if and only if they have the same variables with the same exponents.
The **degree** of a monomial is the sum of the exponents of all of its variables.
The **degree** of a polynomial is the degree of the highest term of the polynomial.
The **coefficient** is the constant part of a term.
The **leading coefficient** is the coefficient of the term of highest degree.
A **zero** of a function, $f(x)$ is a number, a, such that $f(\text{a}) = 0$.
A **root** of a polynomial, $P(x)$, is a number, r, such that $P(\text{r}) = 0$. (A *zero* of the polynomial.)
If $f(x)$ and $g(x)$ are two functions such that $p(x) = f(x)g(x)$, then f and g are **factors** of p.
If a function $p(x)$ can be written as $p(x) = f_1(x)f_2(x)\cdots f_k(x)$, then $f_1(x)f_2(x)\cdots f_k(x)$ is a **factorization** of $p(x)$.
A **linear factor** is a factor of the form $x - \text{a}$, for some constant a.
If a function, $p(x)$, can be factored as $p(x) = (x - \text{a})^k f(x)$ where $x - \text{a}$ is not a factor of $f(x)$, then $x - \text{a}$ is a factor of **multiplicity** k. If k = 1, then $x - \text{a}$ is a **simple factor** of $p(x)$.
***The Remainder Theorem*:** The remainder when dividing a polynomial, $P(x)$, by $x - \text{a}$ is $P(\text{a})$.
The Factor Theorem: $x - \text{a}$ is a factor of polynomial P if and only if $P(\text{a}) = 0$.
The Rational Root Theorem: If $P(x)$ is a polynomial with integer coefficients, then the only possible rational roots of $P(x)$ are of the form:

$$\text{r} = \frac{\text{factor of constant term}}{\text{factor of leading coefficient}}$$

A polynomial is **completely factored** when it is written as a product of linear factors.

Operations with Polynomials

To *add* polynomials, add like terms only.
To *subtract* polynomials, change the signs and add.
To *multiply* two polynomials, use the distributive law and multiply each part of the second polynomial with every part of the first polynomial.

The space of all polynomials is *closed* under addition, subtraction and multiplication. That is, each results in another polynomial.

The Binomial Theorem: The expansion of the binomial $(x + y)^n$ is given by

$$x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + a_3x^{n-3}y^3 + \ldots + a_{n-1}xy^{n-1} + a_ny^n,$$

where each term is of degree n and the coefficients can be calculated as

$$a_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Each term is from the k^{th} position of the n^{th} row of Pascal's Triangle, counting from 0.