Common Core Georgia Performance Standards

Analytic Geometry

Student Resource Book

Unit 7

WALCH EDUCATION

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WALCH EDUCATION

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Introduction

Welcome to the *CCGPS Analytic Geometry Student Resource Book.* This book will help you learn how to use algebra, geometry, data analysis, and probability to solve problems. Each lesson builds on what you have already learned. As you participate in classroom activities and use this book, you will master important concepts that will help to prepare you for the EOCT and for other mathematics assessments and courses.

This book is your resource as you work your way through the Analytic Geometry course. It includes explanations of the concepts you will learn in class; math vocabulary and definitions; formulas and rules; and exercises so you can practice the math you are learning. Most of your assignments will come from your teacher, but this book will allow you to review what was covered in class, including terms, formulas, and procedures.

- In **Unit 1: Similarity, Congruence, and Proofs**, you will learn about dilations, and you will construct lines, segments, angles, polygons, and triangles. You will explore congruence and then define, apply, and prove similarity. Finally, you will prove theorems about lines, angles, triangles, and parallelograms.
- In **Unit 2: Right Triangle Trigonometry**, you will begin by exploring trigonometric ratios. Then you will go on to apply trigonometric ratios.
- In **Unit 3: Circles and Volume**, you will be introduced to circles and their angles and tangents. Then you will learn about inscribed polygons and circumscribed triangles by constructing them and proving properties of inscribed quadrilaterals. You will construct tangent lines and find arc lengths and areas of sectors. Finally, you will explain and apply area and volume formulas.
- In **Unit 4: Extending the Number System**, you will start working with the number system and rational exponents. Then you will perform operations with complex numbers and polynomials.
- In **Unit 5: Quadratic Functions**, you will begin by identifying and interpreting structures in expressions. You will use this information as you learn to create and solve quadratic equations in one variable, including taking the square root of both sides, factoring, completing the square, applying the quadratic formula, and solving quadratic inequalities. You will move on to solving quadratic equations in two or more variables, and solving systems

of equations. You will learn to analyze quadratic functions and to build and transform them. Finally, you will solve problems by fitting quadratic functions to data.

- In **Unit 6: Modeling Geometry**, you will study the links between the two math disciplines, geometry and algebra, as you derive equations of a circle and a parabola. You will use coordinates to prove geometric theorems about circles and parabolas and solve systems of linear equations and circles.

- In **Unit 7: Applications of Probability**, you will explore the idea of events, including independent events, and conditional probability.

Each lesson is made up of short sections that explain important concepts, including some completed examples. Each of these sections is followed by a few problems to help you practice what you have learned. The "Words to Know" section at the beginning of each lesson includes important terms introduced in that lesson.

As you move through your Analytic Geometry course, you will become a more confident and skilled mathematician. We hope this book will serve as a useful resource as you learn.

Lesson 1: Events

Common Core Georgia Performance Standards

MCC9–12.S.CP.1★

MCC9–12.S.CP.2★

MCC9–12.S.CP.7★

Essential Questions

1. How can you describe events?
2. How can you determine whether two events are independent?
3. For events *A* and *B*, how can you find the probability of the event "*A* or *B*"?
4. For events *A* and *B*, how can you find the probability of the event "*A* and *B*"?

WORDS TO KNOW

Addition Rule If *A* and *B* are any two events, then the probability of *A* or *B*, denoted *P*(*A* or *B*), is given by: *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*). Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

complement a set whose elements are not in another set, but are in some universal set being considered. The complement of set *A*, denoted by $\overline{A}$, is the set of elements that are in the universal set, but not in *A*. The event does not occur. The probability of an event not occurring is the one minus the probability of the event occurring, $P\left(\overline{A}\right) = 1 - P(A)$.

dependent events events that are not independent. The outcome of one event affects the probability of the outcome of another event.

disjoint events events that have no outcomes in common. If *A* and *B* are disjoint events, then they cannot both occur. Disjoint events are also called mutually exclusive events.

element an item in a set; also called a member

empty set a set that has no elements, denoted by $\varnothing$. The empty set is also called the null set.

equal sets sets with all the same elements

event	an outcome or set of outcomes of an experiment. An event is a subset of the sample space.
experiment	a process or action that has observable results. The results are called outcomes.
independent events	events such that the outcome of one event does not affect the probability of the outcome of another event
intersection	a set whose elements are each in both of two other sets. The intersection of sets A and B, denoted by $A \cap B$, is the set of elements that are in both A and B.
member	an item in a set; also called an element
mutually exclusive events	events that have no outcomes in common. If A and B are mutually exclusive events, then they cannot both occur. Mutually exclusive events are also called disjoint events.
null set	a set that has no elements, denoted by $\varnothing$. The null set is also called the empty set.
outcome	a result of an experiment
probability	a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur
probability of an event *E*	denoted $P(E)$, and is given by $P(E)=\dfrac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$ in a uniform probability model
probability model	a mathematical model for observable facts or occurrences that are assumed to be random; a representation of a random phenomenon
relative frequency (of an event)	the number of times an event occurs divided by the number of times an experiment is performed
sample space	the set of all possible outcomes of an experiment
set	a collection or list of items
subset	a set whose elements are in another set. Set A is a subset of set B, denoted by $A \subset B$, if all the elements of A are also in B.
uniform probability model	a probability model in which all the outcomes of an experiment are assumed to be equally likely

union — a set whose elements are in at least one of two other sets. The union of sets A and B, denoted by $A \cup B$, is the set of elements that are in either A or B or both A and B.

universal set — a set of all elements that are being considered in a particular situation. In a probability experiment, the universal set is the sample space.

Venn diagram — a diagram that shows how two or more sets in a universal set are related

Recommended Resources

- MathIsFun.com. "Mutually Exclusive Events."

  http://www.walch.com/rr/00091

  This site gives an overview of events that are mutually exclusive and events that are not mutually exclusive, including how to calculate probabilities for both cases. The site explains how the Addition Rule is used and includes practice questions.

- MathIsFun.com. "Probability."

  http://www.walch.com/rr/00092

  This site gives an overview of probability, and includes key definitions and practice questions.

- MathIsFun.com. "Probability: Independent Events."

  http://www.walch.com/rr/00093

  This site reviews dependent and independent events, including how to calculate the probability of two or more independent events. It also has practice questions.

- OnlineMathLearning.com. "Complementary Probability."

  http://www.walch.com/rr/00094

  This website offers an interactive worksheet about complementary probabilities and unions. After completing the worksheet, users can click on the button to check their work. The page also offers hints.

Lesson 7.1.1: Describing Events

Introduction

Probability is a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur. In the study of probability, an **experiment** is any process or action that has observable results. The results are called **outcomes**. The set of all possible outcomes is called the **sample space**. An **event** is an outcome or set of outcomes of an experiment; therefore, an event is a **subset** of the sample space, meaning a set whose elements are in another set, the sample space. In this lesson, you will learn to describe events as subsets of the sample space.

For example, drawing a card from a deck of cards is an experiment. All the cards in the deck are possible outcomes, and they comprise the sample space. The event that a jack of hearts is drawn is a single outcome; it is a set with one element: {jack of hearts}. An **element** is an item or a **member** in a set. The event that a red number card less than 5 is drawn is a set with eight elements: {ace of hearts, ace of diamonds, 2 of hearts, 2 of diamonds, 3 of hearts, 3 of diamonds, 4 of hearts, 4 of diamonds}. Each event is a subset of the sample space.

Note that the words "experiment" and "event" in probability have very specific meanings that are not always the same as the everyday meanings. In the example above, "drawing a card" describes an experiment, not an event. "Drawing a jack of hearts" and "drawing a red number card less than 5" describe events, and those events are sets of outcomes.

Note: In the strictest sense, it can be argued that "drawing a card" does *describe* an event. However, it describes the special-case event that contains all possible outcomes; in other words, it describes the entire sample space. The important distinction to remember is that an event is a set of one or more outcomes, while the action or process is the experiment.

Key Concepts

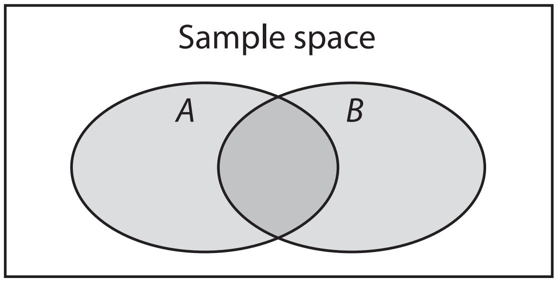
- A **set** is a list or collection of items. Set A is a subset of set B, denoted by $A \subset B$, if all the elements of A are also in B. For example, if $B = \{1, 2, 3, x\}$, then some subsets of B are $\{1, 2, 3\}$, $\{2, x\}$, $\{3\}$, and $\{1, 2, 3, x\}$. Note that a set is a subset of itself.
- An **empty set** is a set that has no elements. An empty set is denoted by $\varnothing$. An empty set is also called a **null set**.

- **Equal sets** are sets with all the same elements. For example, consider sets A, B, and C as follows:
  - A is the set of integers between 0 and 6 that are not odd.
  - B is the set of even positive factors of 4.
  - $C = \{2, 4\}$
- $A = B = C$ because they all have the same elements, 2 and 4.
- The **union** of sets A and B, denoted by $A \cup B$, is the set of elements that are in either A or B or both A and B.
- An **intersection** is a set whose elements are each in both of two other sets. The intersection of sets A and B, denoted by $A \cap B$, is the set of elements that are in both A and B.
- For example, if $A = \{a, b, c\}$ and $B = \{m, a, t, h\}$, then:
  - $A \cup B = \{a, b, c, m, t, h\}$
  - $A \cap B = \{a\}$
- An experiment is any process or action that has observable results. Some examples are:
  - an action, such as drawing a card from a deck or tossing a coin
  - a survey in which participants are asked questions
  - a study, such as taking note of what model vehicles are on a street or how many manufactured items are defective
- The results of an experiment are called outcomes. The set of all possible outcomes is called the sample space.
- As an example, consider experiment #1: toss a coin. The sample space for experiment #1 is {H, T}, where H indicates heads and T indicates tails. Now consider experiment #2: toss a coin and then toss the coin again. The sample space for experiment #2 is {HH, HT, TH, TT}. Note that HT and TH are different outcomes because HT indicates tossing a head and then a tail, whereas TH indicates tossing a tail and then a head.
- An event is an outcome or set of outcomes, so an event is a subset of the sample space.
- Consider this experiment: roll a 6-sided die.
  - The sample space is {1, 2, 3, 4, 5, 6}.

- The event "Roll a number greater than 2" is {3, 4, 5, 6}.
- The event "Roll an odd number less than 3" is {1}.

- The **complement** of set A, denoted by $\overline{A}$, is the set of elements that are in some universal set, but not in A. The complement of A is the event that does not occur.
- A **universal set** is a set of all elements that are being considered in a particular situation. In a probability experiment, the universal set is the sample space.
- For a probability experiment, the complement of event A is the set of all outcomes in the sample space that are not in A. Consider this experiment again: roll a 6-sided die. The sample space is {1, 2, 3, 4, 5, 6}. If event A is described as "roll a number greater than 2," then A = {3, 4, 5, 6} and $\overline{A}$ = {1, 2}.
- When two or more objects are used in an experiment, those objects are distinct, even if they have identical appearances. For example, suppose an experiment consists of rolling two dice and you are considering the event "roll a 2 and a 3." There are two ways to roll a 2 and a 3; that is, the event consists of two outcomes. To understand why, assume that one die is red and the other is blue. You can roll a red 2 and a blue 3 or you can roll a blue 2 and a red 3. Or, if the dice are the same color, you can take a marker and label them "first die" and "second die." Then, after the dice are rolled you can distinguish the outcome "first die 2 and second die 3" from the outcome "first die 3 and second die 2."
- So, when an experiment consists of an action performed with two or more objects at the same time, you can think of the experiment as two or more separate actions with one object. Some examples:
  - Tossing two coins: You can think of this as tossing one coin two times. So the outcomes HT and TH are different. These outcomes comprise the event "toss one head and one tail."
  - Tossing three coins: You can think of this as tossing one coin three times. So the outcomes HHT, HTH, and THH are all different. These outcomes comprise the event "toss exactly two heads."
  - Rolling a pair of dice: You can think of this as rolling one die two times. So the outcome of rolling a 1 and then a 2 is different from the outcome of rolling a 2 and then a 1. These outcomes comprise the event "roll a sum of 3."
- Sometimes it is helpful to draw tables or diagrams to visualize outcomes and the relationships between events.

- A **Venn diagram** is a diagram that shows how two or more sets in a universal set are related. In this diagram, members in event *A* also fit the criteria for members in event *B*, so the circles overlap:



Guided Practice 7.1.1

Example 1

The table that follows shows a group of students and the extra-curricular activities in which they participate.

Student	Chess club	Debate club	Band	German club
Asher			✔	
Ray				
Eva		✔		✔
Merida			✔	
Ysabel		✔	✔	
Ben	✔			
Nate	✔			✔

A student is chosen from the group at random. List the sample space and then describe each of the following events.

{Eva, Nate}   {Ysabel}   {Ben, Nate, Eva}   {Ray, Eva, Ben, Nate}

1. The sample space is the set of all possible outcomes.

   The sample space is {Asher, Ray, Eva, Merida, Ysabel, Ben, Nate}.

2. Describe the event {Eva, Nate}.

   Eva is in the debate club and the German club.

   Nate is in the chess club and the German club.

   To write a description of {Eva, Nate}, consider using the word "or." The activities mentioned are: debate club, German club, and chess club. You might consider this description: "in the debate club, German club, or chess club." But that description would also include Ysabel, who is in the debate club, and Ben, who is in the chess club. So that description is not correct.

   Consider using the word "and." Identify the activity or activities Eva and Nate have in common. They are both in the German club. There are no other students in the German club, so the event can be described as "in the German club."

   A correct description is "the chosen student is in the German club."

3. Describe the event {Ysabel}.

   Ysabel is in the debate club and the band, and there are no other students in both of those activities.

   Therefore, a correct description is "the chosen student is in the debate club and the band."

4. Describe the event {Ben, Nate, Eva}.

Ben is in the chess club.

Nate is in the chess club and the German club.

Eva is in the debate club and the German club.

There is no activity common to all three students, so the description will not use the word "and."

Consider using the word "or." The activities mentioned are: chess club, German club, and debate club. You might consider this description: "in the chess club, German club, or debate club." But that description would also include Ysabel, who is in the debate club. So that description is not correct.

A correct description is "the chosen student is in the chess club or German club."

5. Describe the event {Ray, Eva, Ben, Nate}.

Ray is in no activity.

Eva is in the debate club and the German club.

Ben is in the chess club.

Nate is in the chess club and the German club.

Since Ray is in no activity, consider using the word "not."
Identify any activity that none of the four students are in.

A correct description is "the chosen student is not in the band."

Example 2

Some students were asked what pets they have at home. The following table shows the results of the survey, with the students identified by numbers.

Student	Dog	Cat	Hamster	Bird	Fish
1					
2	✔	✔			
3					
4	✔				
5			✔		✔
6					✔
7	✔	✔		✔	
8					
9		✔		✔	
10	✔	✔			

A student is chosen from the group at random. Consider the following events.

D: The student has a dog.

C: The student has a cat.

H: The student has a hamster.

B: The student has a bird.

F: The student has a fish.

Describe each of the following events by listing outcomes.

C $\quad D \cap B$ $\quad H \cup F$ $\quad \overline{B}$ $\quad \overline{D \cap C}$ $\quad \overline{D \cup C}$

1. List the outcomes of *C*.

   Identify the students who have a cat. List those student numbers in set notation.

   $C = \{2, 7, 9, 10\}$

2. List the outcomes of $D \cap B$.

Identify the students who have a dog. List those student numbers in set notation.

$D = \{2, 4, 7, 10\}$

Identify the students who have a bird. List those student numbers in set notation.

$B = \{7, 9\}$

$D \cap B$ is the intersection of events D and B or the student(s) who have both a dog and a bird. Identify the outcome(s) common to both events.

$D \cap B = \{7\}$

3. List the outcomes of $H \cup F$.

Identify the students who have a hamster. List those student numbers in set notation.

$H = \{5\}$

Identify the students who have a fish. List those student numbers in set notation.

$F = \{5, 6\}$

$H \cup F$ is the union of events H and F or the students who have a hamster or a fish or who have both. Identify the outcomes that appear in either event or both events.

$H \cup F = \{5, 6\}$

4. List the outcomes of $\overline{B}$.

Identify the students who have a bird. List those student numbers in set notation.

$B = \{7, 9\}$

Sample space = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$\overline{B}$ is the set of all outcomes that are in the sample space, but not in B. In other words, $\overline{B}$ is all the students who don't have a bird.

$\overline{B} = \{1, 2, 3, 4, 5, 6, 8, 10\}$

5. List the outcomes of $\overline{D \cap C}$.

To find the outcomes of $\overline{D \cap C}$, we must first determine the outcomes of $D \cap C$.

From step 2, we determined the outcomes in D to be as follows:

$D = \{2, 4, 7, 10\}$

From step 1, we determined the outcomes in C to be as follows:

$C = \{2, 7, 9, 10\}$

The intersection of D and C, then, is the outcome(s) that are in both D and C.

$D \cap C = \{2, 7, 10\}$

$\overline{D \cap C}$ is the set of all outcomes that are in the sample space, but not in $D \cap C$.

$\overline{D \cap C} = \{1, 3, 4, 5, 6, 8, 9\}$

6. List the outcomes of $\overline{D \cup C}$.

First, determine the outcomes of $D \cup C$.

$D = \{2, 4, 7, 10\}$

$C = \{2, 7, 9, 10\}$

$D \cup C = \{2, 4, 7, 9, 10\}$

$\overline{D \cup C}$ is the set of all outcomes that are in the sample space, but not in $D \cup C$.

$\overline{D \cup C} = \{1, 3, 5, 6, 8\}$

Example 3

Hector has entered the following names in the contact list of his new cell phone: Alicia, Brisa, Steve, Don, and Ellis. He chooses one of the names at random to call. Consider the following events.

B: The name begins with a vowel.

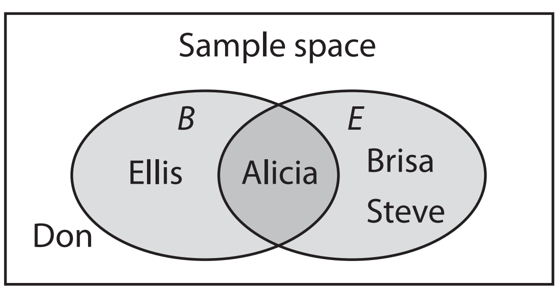
E: The name ends with a vowel.

Draw a Venn diagram to show the sample space and the events *B* and *E*. Then describe each of the following events by listing outcomes.

B $\quad$ E $\quad$ $B \cap E$ $\quad$ $B \cup E$ $\quad$ $\overline{B}$ $\quad$ $\overline{B \cup E}$

1. Draw a Venn diagram. Use a rectangle for the sample space. Use circles or elliptical shapes for the events *B* and *E*.

   Write the students' names in the appropriate sections to show what events they are in.



2. List the outcomes of *B*.

   B = {Ellis, Alicia}

3. List the outcomes of *E*.

   E = {Alicia, Brisa, Steve}

4. List the outcomes of $B \cap E$.

   B = {Ellis, Alicia}

   E = {Alicia, Brisa, Steve}

   $B \cap E$ is the intersection of events *B* and *E*. Identify the outcome(s) common to both events.

   $B \cap E$ = {Alicia}

5. List the outcomes of $B \cup E$.

   B = {Ellis, Alicia}

   E = {Alicia, Brisa, Steve}

   $B \cup E$ is the union of events *B* and *E*. Identify the outcomes that appear in either event or both events.

   $B \cup E$ = {Ellis, Alicia, Brisa, Steve}

6. List the outcomes of $\overline{B}$.

   B = {Ellis, Alicia}

   Sample space = {Alicia, Brisa, Steve, Don, Ellis}

   $\overline{B}$ is the set of all outcomes that are in the sample space, but not in *B*.

   $\overline{B}$ = {Brisa, Steve, Don}

7. List the outcomes of $\overline{B \cup E}$.

B = {Ellis, Alicia}

E = {Alicia, Brisa, Steve}

$B \cup E$ = {Ellis, Alicia, Brisa, Steve}

Sample space = {Alicia, Brisa, Steve, Don, Ellis}

$\overline{B \cup E}$ is the set of all outcomes that are in the sample space, but not in $B \cup E$.

$\overline{B \cup E}$ = {Don}

Example 4

An experiment consists of tossing a coin three times. Consider the following events.

A: The first toss is heads.

B: The second toss is heads.

C: There are two consecutive heads.

D: There are two consecutive tails.

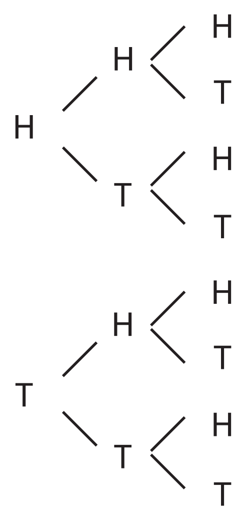
E: The first toss is heads and the second toss is heads.

F: There are neither two consecutive heads nor two consecutive tails.

List the sample space. Then express events *E* and *F* in terms of other events and list the outcomes of *E* and *F*.

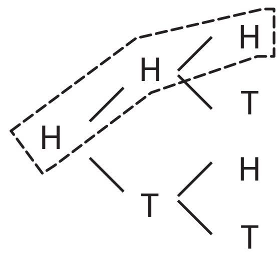
1. The sample space is the set of all possible outcomes.

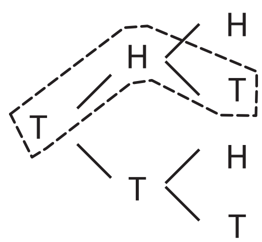
   The tree diagram below shows all possible outcomes, using H for heads and T for tails.



(*continued*)

To identify all the possible outcomes, trace all the different paths from left to right. The following diagram shows two different paths. The first indicated path identifies the outcome HHH, which means heads on all three coin tosses. The second indicated path identifies the outcome THT, which means tails, then heads, then tails.





There are eight different paths, indicating the following sample space:

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. To express event E in terms of other events, first note how the descriptions of A, B, and E are related.

A: The first toss is heads.

B: The second toss is heads.

E: The first toss is heads and the second toss is heads.

Using the word "and" in this way indicates that E is the intersection of A and B.

$$E = A \cap B$$

To list the outcomes of E, list all the outcomes that are in both A and B.

$$E = A \cap B = \{\text{HHH, HHT}\}$$

3. To express event F in terms of other events, first note how the descriptions of C, D, and F are related.

   C: There are two consecutive heads.

   D: There are two consecutive tails.

   F: There are neither two consecutive heads nor two consecutive tails.

   The union of C and D, denoted $C \cup D$, is the event that there are either two consecutive heads or two consecutive tails.

   $C \cup D$ = {HHH, HHT, HTT, THH, TTH, TTT}

   F is the event that there are *neither* two consecutive heads *nor* two consecutive tails. So F is the complement of $C \cup D$.

   $F = \overline{C \cup D}$

   To list the outcomes of F, list all the outcomes that are in the sample space, but not in $C \cup D$.

   Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

   $F = \overline{C \cup D}$ = {HTH, THT}

Example 5

An experiment consists of rolling a pair of dice. How many ways can you roll the dice so that the product of the two numbers rolled is less than their sum?

1. Begin by showing the sample space.

   This diagram of ordered pairs shows the sample space.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

   Key: (2, 3) means 2 on the first die and 3 on the second die.

2. Identify all the outcomes in the event "the product is less than the sum."

For (1, 1), the product is $1 \times 1 = 1$ and the sum is $1 + 1 = 2$, so the product is less than the sum.

For (1, 2), the product is $1 \times 2 = 2$ and the sum is $1 + 2 = 3$, so the product is less than the sum.

This table shows all the possible products and sums.

Outcome	**Product**	**Sum**	**Product < sum**	**Outcome**	**Product**	**Sum**	**Product < sum**
(1, 1)	1	2	Yes	(4, 1)	4	5	Yes
(1, 2)	2	3	Yes	(4, 2)	8	6	No
(1, 3)	3	4	Yes	(4, 3)	12	7	No
(1, 4)	4	5	Yes	(4, 4)	16	8	No
(1, 5)	5	6	Yes	(4, 5)	20	9	No
(1, 6)	6	7	Yes	(4, 6)	24	10	No
(2, 1)	2	3	Yes	(5, 1)	5	6	Yes
(2, 2)	4	4	No	(5, 2)	10	7	No
(2, 3)	6	5	No	(5, 3)	15	8	No
(2, 4)	8	6	No	(5, 4)	20	9	No
(2, 5)	10	7	No	(5, 5)	25	10	No
(2, 6)	12	8	No	(5, 6)	30	11	No
(3, 1)	3	4	Yes	(6, 1)	6	7	Yes
(3, 2)	6	5	No	(6, 2)	12	8	No
(3, 3)	9	6	No	(6, 3)	18	9	No
(3, 4)	12	7	No	(6, 4)	24	10	No
(3, 5)	15	8	No	(6, 5)	30	11	No
(3, 6)	18	9	No	(6, 6)	36	12	No

By checking all the outcomes in the sample space, you can verify that the product is less than the sum for only these outcomes:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (3, 1) (4, 1) (5, 1) (6, 1)

3. Count the outcomes that meet the event criteria.

There are 11 ways to roll two dice so that the product is less than the sum.

Practice 7.1.1: Describing Events

A real estate agent compiled data on some houses in a particular neighborhood. The table below shows his data. Use the table to complete problems 1 and 2.

House number	Fireplace	Oil heat	Garage	Central A/C
202		✔	✔	
204		✔		✔
206	✔	✔	✔	✔
208				
210	✔	✔		

1. A house is chosen from the group at random. Describe the event {206, 210}.

2. A house is chosen from the group at random. Describe the event {202, 206, 210}.

Some students were asked what states they have visited in the last two years. The table shows the results of the survey. Use the table and the information on the next page to complete problems 3 and 4.

Student	Florida	Texas	South Carolina	Alabama	California
Monica					
Opal		✔			
Colin	✔	✔	✔		
Josh					
Elias					
Hugo	✔		✔		✔
Carla					✔
Sam					
Rick	✔		✔		✔

continued

A student is chosen from the group at random. Consider the following events.

FL: The student has visited Florida.

TX: The student has visited Texas.

SC: The student has visited South Carolina.

AL: The student has visited Alabama.

CA: The student has visited California.

3. List the outcomes of $TX \cup CA$.

4. List the outcomes of $\overline{FL \cap SC}$.

Some cars in a school parking lot have the following license plate identifications. Use this list and the information that follows to complete problems 5 and 6.

MRB 5516 KXL 2677 JEV 8591 AAR 8214 BCF 4435 XTE 6165 LSU 7582

One of the license plates is chosen at random. Consider the following events.

V: The chosen license plate has at least one vowel.

S: The sum of the digits on the chosen license plate is greater than 20.

5. Draw a Venn diagram to show the sample space and the events *V* and *S*.

6. Describe the events $\overline{V}$ and $V \cap S$ by listing their outcomes.

continued

Use the given information to complete problems 7 and 8.

Jim, Palo, and Marigold work at a coffee shop. On Monday, the boss assigns one of them at random to roast the coffee beans. Then on Tuesday, the boss again assigns one of them at random to roast the coffee beans.

Consider the following events.

A: Marigold is assigned twice.

B: The same worker is assigned twice.

C: Neither Jim nor Palo is assigned.

D: Two different workers are assigned.

7. Choosing from *B*, *C*, and *D*, what event(s) are equal to *A*?

8. Choosing from *A*, *B*, and *C*, what event(s) are equal to $\overline{D}$?

For problems 9 and 10, consider the experiment of rolling a pair of dice.

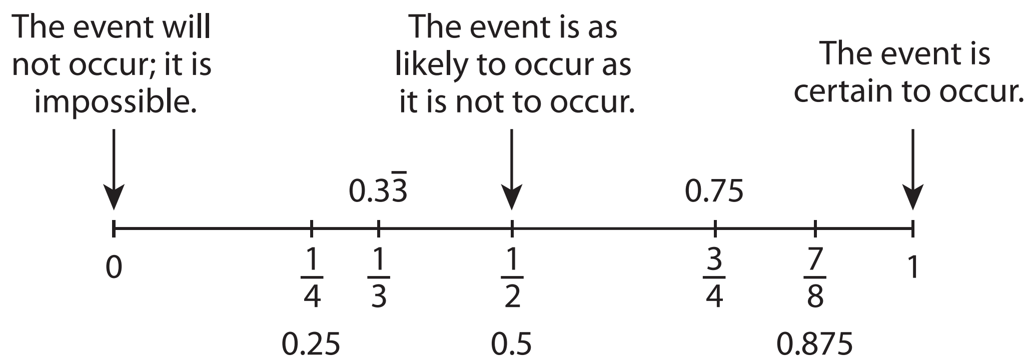
9. How many ways can you roll a pair of dice and get a sum greater than 8?

10. How many ways can you roll a pair of dice and get a difference less than or equal to 2?

Lesson 7.1.2: The Addition Rule

Introduction

Remember that probability is a number from 0 to 1 inclusive or a percent from 0% to 100% inclusive that indicates how likely an event is to occur. A probability of 0 indicates that the event cannot occur. A probability of 1 indicates that the event is certain to occur. The following diagram shows some sample probabilities.



When all the outcomes of an experiment are equally likely, the **probability of an event *E***, denoted *P*(*E*), is given by $P(E)=\frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.

For situations involving more than one event, the Addition Rule sometimes applies. You will learn about this rule.

Key Concepts

- Probability is the likelihood of an event.
- A **probability model** is a mathematical model for observable facts or occurrences that are assumed to be random. The facts or occurrences are called outcomes. Actions and processes that produce outcomes are called experiments.
- When all the outcomes of an experiment are assumed to be equally likely, then the probability model is a **uniform probability model**.
- In a uniform probability model, the probability of an event *E*, denoted *P*(*E*), is given by the formula $P(E)=\frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.

- The frequency of an event is the number of times it occurs. The **relative frequency** of an event is the number of times it occurs divided by the number of times the experiment is performed.
- The probability of an event can be used to predict its relative frequency if the experiment is performed a large number of times. For example, the probability of getting a 3 by rolling a fair die is $\frac{1}{6}$. So if you roll a fair die 600 times, it is reasonable to predict that you will get a 3 about 100 times, or $\frac{1}{6}$ of the number of times you roll the die.
- According to the **Addition Rule,** if *A* and *B* are any two events, then the probability of *A* or *B*, denoted *P*(*A* or *B*), is given by the formula *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*).
- Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- The Addition Rule contains four probabilities: *P*(*A* or *B*), *P*(*A*), *P*(*B*), and *P*(*A* and *B*); if three of them are known, then the equation can be solved to find the fourth.
- If events *A* and *B* have no outcomes in common, then they are **mutually exclusive events**, also known as **disjoint events**. If *A* and *B* are mutually exclusive events, then they cannot both occur, so *P*(*A* and *B*) = 0.
- This leads to the following special case of the Addition Rule for mutually exclusive events:

  P(*A* or *B*) = *P*(*A*) + *P*(*B*), or $P(A \cup B) = P(A) + P(B)$

Guided Practice 7.1.2

Example 1

Bobbi tosses a coin 3 times. What is the probability that she gets exactly 2 heads? Write your answer as a fraction, as a decimal, and as a percent.

1. Identify the sample space and count the outcomes.

   Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

   There are 8 outcomes in the sample space.

2. Identify the outcomes in the event and count the outcomes.

   There are 3 outcomes in the event "exactly 2 heads:" HHT, HTH, and THH.

3. Apply the formula for the probability of an event.

   Divide the number of outcomes for the event Bobbi is hoping for (3) by the total number of possible outcomes (8).

   $$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$$

   $$P(\text{exactly 2 heads}) = \frac{3}{8} = 0.375 = 37.5\%$$

Example 2

Donte is playing a card game with a standard 52-card deck. He's hoping for a club or a face card on his first draw. What is the probability that he draws a club or a face card on his first draw?

1. Identify the sample space and count the outcomes.

   The sample space is the set of all cards in the deck, so there are 52 outcomes.

2. Identify the outcomes in the event and count the outcomes.

You can use a table to show the sample space. Then identify and count the cards that are either a club or a face card or both a club and a face card.

Suit	2	3	4	5	6	7	8	9	10	J	Q	K	A
Spade										✔	✔	✔	
Club	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
Diamond										✔	✔	✔	
Heart										✔	✔	✔	

The event "club or face card" has 22 outcomes.

3. Apply the formula for the probability of an event.

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$$

$$P(\text{club or face card}) = \frac{22}{52} = \frac{11}{26} \approx 0.42$$

4. Apply the Addition Rule to verify your answer.

Let *A* be the event "club." There are 13 clubs, so *A* has 13 outcomes.

Let *B* be the event "face card." There are 12 face cards, so *B* has 12 outcomes.

The event "*A* and *B*" is the event "club and face card," which has 3 outcomes: jack of clubs, queen of clubs, and king of clubs.

Apply the Addition Rule.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{club or face card}) = P(\text{club}) + P(\text{face card}) - P(\text{club and face card})$

$$P(\text{club or face card}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26} \approx 0.42$$

The Addition Rule answer checks out with the probability found in step 3, so the probability of the event is approximately 0.42.

Example 3

Corrine is playing a board game. To find the number of spaces to move, she rolls a pair of dice. On her next roll she wants doubles or a sum of 10. What is the probability that she rolls doubles or a sum of 10 on her next roll? Interpret your answer in terms of a uniform probability model.

1. Identify the sample space and count the outcomes.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

There are 36 outcomes in the sample space.

2. Apply the Addition Rule to find the probability that she rolls doubles or a sum of 10 on her next roll.

Let A be the event "doubles." Event A has 6 outcomes: (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6).

Let B be the event "sum of 10." Event B has 3 outcomes: (4, 6), (5, 5), and (6, 4).

The event "A and B" is the event "doubles and sum of 10," which has 1 outcome: (5, 5).

Apply the Addition Rule.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(\text{doubles or sum of 10}) = P(\text{doubles}) + P(\text{sum of 10}) - P(\text{doubles and sum of 10})$$

$$P(\text{doubles or sum of 10}) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9} \approx 0.22 \approx 22\%$$

3. Interpret the answer in terms of a uniform probability model.

The probabilities used in the application of the Addition Rule, $\frac{6}{36}$, $\frac{3}{36}$, and $\frac{1}{36}$, are found by using the formula $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.

This formula is a uniform probability model, which requires that all outcomes in the sample space be equally likely. It is reasonable to assume that the dice Corrine rolls are fair, so that all outcomes in the sample space are equally likely. Therefore, the answer is valid and can serve as a reasonable predictor. You can predict that the relative frequency of getting doubles or a sum of 10 will be about 22% for a large number of dice rolls. That is, you can predict that Corrine will get doubles or a sum of 10 about 22% of the time if she rolls the dice a large number of times.

Example 4

Students at Rolling Hills High School receive an achievement award for either performing community service or making the honor roll. The school has 500 students and 180 of them received the award. There were 125 students who performed community service and 75 students who made the honor roll. What is the probability that a randomly chosen student at Rolling Hills High School performed community service and made the honor roll?

1. Define the sample space and state its number of outcomes.

The sample space is the set of all students at the school; it has 500 outcomes.

2. Define events that are associated with the numbers in the problem and state their probabilities.

Let *A* be the event "performed community service." Then $P(A)=\frac{125}{500}$.

Let *B* be the event "made the honor roll." Then $P(B)=\frac{75}{500}$.

The event "*A* or *B*" is the event "performed community service or made the honor roll," and can also be written $A\cup B$. $P(A\cup B)=\frac{180}{500}$ because 180 students received the award for either community service or making the honor roll.

3. Write the Addition Rule and solve it for *P*(*A* and *B*), which is the probability of the event "performed community service and made the honor roll." *P*(*A* and *B*) can also be written $P(A\cap B)$.

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$\frac{180}{500}=\frac{125}{500}+\frac{75}{500}-P(A\cap B)$ Substitute the known probabilities.

$\frac{180}{500}=\frac{200}{500}-P(A\cap B)$ Simplify.

$-\frac{20}{500}=-P(A\cap B)$ Subtract $\frac{200}{500}$ from both sides.

$\frac{20}{500}=P(A\cap B)$ Multiply both sides by –1.

The probability that a randomly chosen student at Rolling Hills High School has performed community service and is on the honor roll is $\frac{20}{500}=\frac{1}{25}$, or 4%.

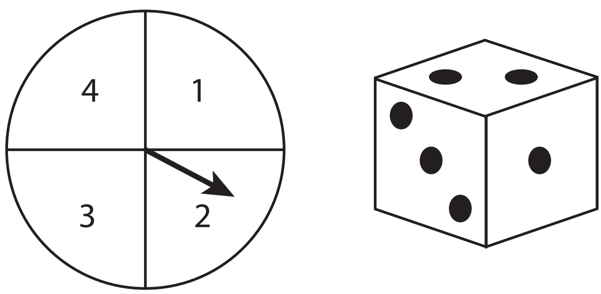
Practice 7.1.2: The Addition Rule

Use what you have learned about probability to solve the following problems.

1. Giada tosses a coin 3 times. What is the probability that she gets at least 2 consecutive heads or at least 2 consecutive tails? Write your answer as a fraction, as a decimal, and as a percent.

Use the following diagram and information to complete problems 2 and 3.

Walton is playing a board game. He spins the spinner and rolls a standard 6-sided die. Let *s* be the spinner result and let *d* be the die result.



2. What is the probability that $s + d > 5$ or $s \bullet d > 5$?

3. What is the probability that $s + d > 5$ and $s \bullet d < 10$?

4. Elvio is playing a card game with a standard 52-card deck. He wants a red card or a face card on his first draw. What is the probability that he gets a red card or a face card on his first draw?

continued

Vick is playing a board game. To find the number of spaces to move, he rolls a pair of dice. Use this information to complete problems 5 and 6.

5. What is the probability that Vick rolls doubles or a sum of 2? Write your answer as a fraction, as a decimal, and as a percent.

6. What is the probability that Vick rolls the dice so that both numbers are even and one number is twice the other number?

Ridgeview High School has 206 students in the eleventh grade. Use this information to complete problems 7 and 8.

7. The only eleventh grade art courses are drawing and painting. There are 75 eleventh grade students taking at least one art course. There are 35 taking drawing and 40 taking painting. What is the probability that a randomly chosen eleventh grader is taking both drawing and painting?

8. The only eleventh grade physical education courses are weight training and team sports. There are 195 eleventh grade students taking at least one physical education course. There are 82 taking weight training and 120 taking team sports. What is the probability that a randomly chosen eleventh grader is taking both weight training and team sports?

continued

Use the information provided below to complete problems 9 and 10.

The table below lists the first name and locker combination for five students. The information is presented in letters (for the names) and digits (for the combinations). The total characters for any given student are both letters and digits in their data. The characters of the names are not case sensitive. For example, Jamal's characters are j, a, m, a, l, 6, 3, and 2.

Student	Combination
Jamal	6–3–2
Crystal	25–6–27
Doug	2–8–12
Leslie	6–4–8
Zahara	23–22–5

9. If a student is chosen at random from the group, what is the probability that the choice will contain a repeated letter and a repeated digit?

10. If a student is chosen at random from the group, what is the probability that the choice will contain fewer than 10 characters or a digit sum less than 20?

Lesson 7.1.3: Understanding Independent Events

Introduction

In probability, events are either dependent or independent. Two events are **independent** if the occurrence or non-occurrence of one event has no effect on the probability of the other event. If two events are independent, then you can simply multiply their individual probabilities to find the probability that both events will occur. If events are **dependent**, then the outcome of one event affects the outcome of another event. So it is important to know whether or not two events are independent.

Two events *A* and *B* are independent if and only if *P*(*A* and *B*) = *P*(*A*) • *P*(*B*). But sometimes you only know two of these three probabilities and you want to find the third. In such cases, you can't test for independence, so you might assess the situation or nature of the experiment and then make an assumption about whether or not the events are independent. Then, based on your assumption, you can find the third probability in the equation.

Key Concepts

- Two events *A* and *B* are independent if and only if they satisfy the following test:

  $P(A \text{ and } B) = P(A) \bullet P(B)$

- Using set notation, the test is $P(A \cap B) = P(A) \bullet P(B)$.
- Sometimes it is useful or necessary to make an assumption about whether or not events are independent, based on the situation or nature of the experiment. Then any conclusions or answers are based on the assumption.
- In a uniform probability model, all the outcomes of an experiment are assumed to be equally likely, and the probability of an event *E*, denoted *P*(*E*), is given by $P(E) = \dfrac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.
- When this definition of probability is used and the relevant probabilities are known, then the definition of independent events can be used to determine, verify, or prove that two events are dependent or independent.
- The relative frequency of an event is the number of times it occurs divided by the number of times the experiment is performed (called trials) or the number of observations:

  $$\text{relative frequency} = \frac{\text{number of times event occurs}}{\text{number of trials or observations}}$$

- Relative frequency can be used to estimate probability in some cases where a uniform probability model does not seem appropriate. Such cases include data collected by surveys or obtained by observations.
- When relative frequency is used to estimate the relevant probabilities, then the definition of independent events can be used to determine whether two events *seem to be* dependent or independent, *based on the data*.
- Probability and relative frequency are related as follows:
  - The probability of an event can be used to predict its relative frequency if the experiment is performed a large number of times. For example, the probability of getting a 3 by rolling a fair die is $\frac{1}{6}$. So if you roll a fair die 6,000 times, it is reasonable to predict that you will get a 3 about 1,000 times, or $\frac{1}{6}$ of the number of times you roll the die. But a prediction is not a guarantee. If you roll a die 6,000 times, the number of times you get a 3 might not be 1,000.
  - Relative frequency can be used to predict the probability of an event. In general, as the number of trials or observations increases, the prediction becomes stronger. For example, suppose you ask 40 people for their favorite ice cream flavor. If 8 say chocolate, then you can predict that the probability of a randomly selected person saying chocolate is $\frac{8}{40}$, or 20%. But 40 people make up a small sample, so this is not a very strong prediction. Now suppose you ask 4,000 people who are randomly selected using a good sampling method. If 740 say chocolate, then you can predict that the probability of a randomly selected person saying chocolate is $\frac{740}{4000}$, or 18.5%; this is a stronger prediction.
- Also remember the Addition Rule: If A and B are any two events, then the probability of A or B, denoted $P(A \text{ or } B)$, is given by $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- Using set notation, the rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Guided Practice 7.1.3

Example 1

Ms. Martinez is teaching her students about probability. She has a bag of 6 marbles; 2 are red and 4 are black. She asked her students to conduct the following experiments.

Experiment A: Pick a random marble from the bag, then *replace it*, and then pick a random marble from the bag again.

Experiment B: Pick a random marble from the bag, *do not replace it*, and then pick a random marble from the bag again.

For each experiment, determine whether getting red on the first pick and getting black on the second pick are independent events.

1. Assign variables as names of the events.

   Let *RF* be the event that a red marble is picked first.

   Let *BS* be the event that a black marble is picked second.

2. For experiment A, determine whether *RF* and *BS* are independent events.

   In experiment A, the marble is *replaced* in the bag after the first pick, so the conditions are identical on the first and second picks. That is, for both picks, the bag has 2 red marbles and 4 black marbles. Whether event *RF* does or does not occur has no effect on the probability that *BS* occurs.

   So for experiment A, *RF* and *BS* are independent events.

3. For experiment B, determine whether *RF* and *BS* are independent events.

   In experiment B, the marble is *not replaced* in the bag after the first pick, so the conditions are different on the first and second picks. There are 6 marbles in the bag for the first pick and 5 marbles in the bag for the second pick. If event *RF* occurs, then for the second pick the bag will have 1 red marble and 4 black marbles. If event *RF* does not occur, then for the second pick the bag will have 2 red marbles and 3 black marbles. So whether event *RF* does or does not occur does affect the probability of *BS*.

   Therefore, for experiment B, *RF* and *BS* are dependent events.

Example 2

Trevor tosses a coin 3 times. Consider the following events.

A: The first toss is heads.

B: The second toss is heads.

C: There are exactly 2 consecutive heads.

For each of the following pairs of events, determine if the events are independent.

A and B (This is $A \cap B$ in set notation.)

A and C (This is $A \cap C$ in set notation.)

B and C (This is $B \cap C$ in set notation.)

1. List the sample space.

   Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. Use the sample space to determine the relevant probabilities.

$P(A) = \frac{4}{8} = \frac{1}{2}$	There are 4 outcomes with heads first.
$P(B) = \frac{4}{8} = \frac{1}{2}$	There are 4 outcomes with heads second.
$P(C) = \frac{2}{8} = \frac{1}{4}$	There are 2 outcomes with exactly 2 consecutive heads.
$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$	There are 2 outcomes with heads first and heads second.
$P(A \cap C) = \frac{1}{8}$	There is 1 outcome with heads first and exactly 2 consecutive heads.
$P(B \cap C) = \frac{2}{8} = \frac{1}{4}$	There are 2 outcomes with heads second and exactly 2 consecutive heads.

3. Use the definition of independence to determine if the events are independent in each specified pair.

$$P(A \cap B) = P(A) \bullet P(B)$$

$$\frac{1}{4} = \frac{1}{2} \bullet \frac{1}{2}$$ *A* and *B* are independent.

$$P(A \cap C) = P(A) \bullet P(C)$$

$$\frac{1}{8} = \frac{1}{2} \bullet \frac{1}{4}$$ *A* and *C* are independent.

$$P(B \cap C) = P(B) \bullet P(C)$$

$$\frac{1}{4} \neq \frac{1}{2} \bullet \frac{1}{4}$$ *B* and *C* are dependent.

Example 3

Landen owns a delicatessen. He collected data on sales of his most popular sandwiches for one week and recorded it in the table below.

Sandwiches Sold in One Week

Bread choice	**Sandwich choice**			
	Landen's club	Turkey melt	Roasted chicken	Veggie delight
Country white	44	25	25	8
Whole wheat	24	28	26	34
Sourdough	24	27	24	31

Each of the following statements describes a pair of events. For each statement, determine if the events seem to be independent based on the data in the table.

A random customer orders Landen's club sandwich on country white bread.

A random customer orders the roasted chicken sandwich on whole wheat bread.

1. Find the totals of all the categories.

Bread choice	**Sandwich choice**				**Total**
	Landen's club	Turkey melt	Roasted chicken	Veggie delight	
Country white	44	25	25	8	102
Whole wheat	24	28	26	34	112
Sourdough	24	27	24	31	106
Total	92	80	75	73	320

2. For the first statement, assign variables as names of the events.

   LC: A random customer orders Landen's club sandwich.

   CW: A random customer orders country white bread.

3. Use the data to determine the relevant probabilities.

   $P(LC) = \frac{92}{320}$ There were 320 sandwiches sold, and 92 of them were Landen's club.

   $P(CW) = \frac{102}{320}$ There were 320 sandwiches sold, and 102 of them were on country white bread.

   $P(LC \cap CW) = \frac{44}{320}$ There were 320 sandwiches sold, and 44 of them were Landen's club on country white bread.

4. Use the definition of independence to determine if the events seem to be independent.

   $P(LC \cap CW) = P(LC) \bullet P(CW)$

   $\frac{44}{320} = \frac{92}{320} \bullet \frac{102}{320}$ Substitute probabilities and simplify.

   $0.138 \neq 0.092$ *LC* and *CW* seem to be dependent, based on the data.

5. For the second statement, assign variables as names of the events.

RC: A random customer orders a roasted chicken sandwich.

WW: A random customer orders whole wheat bread.

6. Use the data to determine the relevant probabilities.

$P(RC) = \frac{75}{320}$ There were 320 sandwiches sold, and 75 of them were roasted chicken.

$P(WW) = \frac{112}{320}$ There were 320 sandwiches sold, and 112 of them were on whole wheat bread.

$P(RC \cap WW) = \frac{26}{320}$ There were 320 sandwiches sold, and 26 of them were roasted chicken on whole wheat bread.

7. Use the definition of independence to determine if the events seem to be independent.

$$P(RC \cap WW) = P(RC) \bullet P(WW)$$

$$\frac{26}{320} = \frac{75}{320} \bullet \frac{112}{320}$$ Substitute probabilities and simplify.

$$0.081 \approx 0.082$$

RC and *WW* seem to be independent based on the data.

Example 4

The Town Cinema is next to Rhiannon's Bistro. A consultant surveyed 200 people who saw a particular movie at the Town Cinema and also ate at Rhiannon's Bistro. The survey indicated that 80% liked the movie and 60% liked both the movie and their meal at Rhiannon's Bistro. Assume that liking the movie and liking a meal at Rhiannon's Bistro are independent events. Based on the survey, what is the probability that a randomly chosen moviegoer will like a meal at Rhiannon's Bistro?

1. Define the relevant events and assign them variable names.

   Let M be the event that a randomly chosen person will like the movie.

   Let R be the event that a randomly chosen person will like a meal at Rhiannon's Bistro.

   Therefore, "M and R" is the event that a randomly chosen person will like both the movie and a meal at Rhiannon's Bistro.

2. Assign relevant probabilities based on the survey results.

   $P(M) = 80\%$ — To assign probability based on data, use relative frequency.

   $P(M \text{ and } R) = 60\%$

3. Write the equation contained in the definition of independent events. Solve it for the desired probability.

   $P(M \text{ and } R) = P(M) \bullet P(R)$

   $0.60 = 0.80 \bullet P(R)$ — Substitute probabilities using decimal equivalents of percents.

   $\frac{0.60}{0.80} = P(R)$ — Divide both sides by 0.80.

   $0.75 = P(R)$ — Simplify.

   Based on the survey, the probability that a randomly chosen person will like a meal at Rhiannon's Bistro is 0.75, or 75%.

Example 5

Joel is in a basketball game. He has just tied the game. He was fouled while shooting, so he is awarded a free throw. Moreover, the referee calls a technical foul on the other team because of bad behavior, so Joel's team is awarded a second free throw attempt. The coach assigns a different player, Rico, to shoot the second free throw. Joel has made 32 of 50 free throws so far this season, and Rico has made 42 of 48. What is the probability that Joel or Rico will make a free throw to win the game?

1. Define the relevant events and assign them variable names.

   Let J be the event that Joel makes his free throw.

   Let R be the event that Rico makes his free throw.

   "J or R" is the event that either Joel or Rico makes a free throw.

   "J and R" is the event that both Joel and Rico make a free throw.

2. Assign relevant probabilities based on past performance over the season.

   $P(J) = \frac{32}{50}$ Joel has made 32 of 50 free throws so far this season.

   $P(R) = \frac{42}{48}$ Rico has made 42 of 48 free throws so far this season.

3. Decide if it is reasonable to assume that the events in the problem are independent.

   It is reasonable to assume that the probability of either player making his free throw is not affected by whether the other makes his free throw. Therefore, the events J and R are independent.

4. Use the Addition Rule and the definition of independent events. Solve the equation for the desired probability.

$P(J \text{ or } R) = P(J) + P(R) - P(J \text{ and } R)$	Write the Addition Rule.
$P(J \text{ or } R) = P(J) + P(R) - P(J) \bullet P(R)$	*J* and *R* are independent, so substitute $P(J) \bullet P(R)$ for $P(J \text{ and } R)$.
$P(J \text{ or } R) = \frac{32}{50} + \frac{42}{48} - \left(\frac{32}{50}\right)\left(\frac{42}{48}\right)$	Substitute probabilities.
$P(J \text{ or } R) = 0.64 + 0.875 - (0.64)(0.875)$	Substitute decimal equivalents.
$P(J \text{ or } R) = 0.64 + 0.875 - 0.56$	Simplify, then solve.
$P(J \text{ or } R) = 0.955$	

Based on past performance this season and assuming the events are independent, the probability that either Joel or Rico will make a free throw to win the game is 0.955, or 95.5%.

Practice 7.1.3: Understanding Independent Events

Use what you know about independent events to solve.

1. The table below shows four sets of values for $P(A)$, $P(B)$, and $P(A \cap B)$. Based on the definition of independence, determine if events A and B are independent in each case.

	$P(A)$	$P(B)$	$P(A \cap B)$	Are A and B independent? (yes/no)
a.	0.25	0.40	0.65	
b.	0.42	0.15	0.063	
c.	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{12}$	
d.	$\frac{3}{4}$	$\frac{4}{15}$	$\frac{1}{5}$	

Use the following information to complete problems 2 and 3.

Danitza is playing a card game. She will take cards from a pile of cards that are face down. The pile contains the following 5 cards: 10 of spades, 10 of hearts, jack of spades, jack of diamonds, and ace of clubs. For each of problems 2 and 3, find the probability of getting a heart first and getting a jack second. In each problem, state whether the events are independent.

2. Danitza takes a card, then replaces it, and then takes a second card.

3. Danitza takes a card, does not replace it, and then takes a second card.

continued

For problem 4, suppose that a married couple will have 3 children and suppose that having a boy or girl is equally likely each time. Consider the following events.

A: At least 1 child is a boy.

B: The first child is a boy.

C: The third child is a girl.

4. For each of the following pairs of events, determine if the events are independent.

   A and *B*

   A and *C*

   B and *C*

Use the table and the given information to complete problems 5 and 6.

Three attorneys are partners in a law firm. The table shows data about the attorneys' clients.

Client Education Levels

Attorney	**Client's education level**			
	Less than high school diploma	High school diploma	Four-year college degree	Advanced college degree
Jones	2	8	18	7
Walker	8	20	9	7
Gutierrez	2	10	17	6

Using the data in the table, determine if the events stated in problems 5 and 6 seem to be independent. Show the work that supports your answer.

5. A client is Jones' client and has less than a high school diploma.

6. A client is Walker's client and has an advanced college degree.

continued

Use the given information to solve problems 7–10.

7. A high school counselor surveys a well-chosen sample that represents all the students in the eleventh grade. She finds that 62.5% have a part-time job. Another 25% have a part-time job and also have a first-choice college they would like to attend. Assume that having a part-time job and having a first-choice college are independent events. Based on the survey, what is the probability that a randomly chosen eleventh-grade student at the school has a first-choice college?

8. On a certain day, sixty-five percent of customers at a restaurant choose outdoor seating. Sixty percent order an appetizer. Assume that choosing outdoor seating and ordering an appetizer are independent events. What is the probability that the next customer will choose outdoor seating or order an appetizer, based on the data?

9. A restaurant survey indicates that 90% of the customers approve of the service, and 67.5% of the customers approve of both the service and the food. Assume that approving of the service and approving of the food are independent events. What is the probability that a randomly chosen customer at the restaurant will approve of the food, based on the data?

10. At Val's Lawn and Garden Equipment, every transaction is recorded as either "sale of new item" or "service." Last month, 55% of people who visited Val's spent money there, and 25% of the visitors spent money on service. Assume that buying a new item and paying for service are independent events. What is the probability that the next visitor at Val's will buy a new item, based on last month's data? (*Hint*: Use the Addition Rule.)

Lesson 2: Conditional Probability

Common Core Georgia Performance Standards

MCC9–12.S.CP.3★

MCC9–12.S.CP.4★

MCC9–12.S.CP.5★

MCC9–12.S.CP.6★

Essential Questions

1. What are the different ways to find a conditional probability?
2. How can you use a two-way frequency table to find probabilities and conditional probabilities?
3. What are the different ways to determine if two events are independent?
4. How do you sometimes modify the test for independent events when using real-world data?

WORDS TO KNOW

conditional probability of *B* given *A* — the probability that event B occurs, given that event A has already occurred. If A and B are two events from a sample space with $P(A) \neq 0$, then the conditional probability of B given A, denoted $P(B|A)$, has two equivalent expressions:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\text{number of outcomes in } (A \text{ and } B)}{\text{number of outcomes in } A}.$$

independent events — events such that the outcome of one event does not affect the probability of the outcome of another event

two-way frequency table — a frequency table that shows two categories of characteristics, one in rows and the other in columns. Each cell value is a frequency that shows how many times two different characteristics appear together, or how often characteristics are associated with a person, object, or type of item that is being studied.

Recommended Resources

- Interactivate. “Conditional Probability.”

  http://www.walch.com/rr/00095

  This website presents conditional probability and compares it to simple probability in a question-and-answer format.

- MathIsFun.com. “Conditional Probability.”

  http://www.walch.com/rr/00096

  This site gives an overview of conditional probability and has practice questions.

- StatisticsHelp.com. “Probability: Conditional Probability Demo.”

  http://www.walch.com/rr/00097

  Users of this site can view a sample space and select different options for events to see the conditional probability of that event within the given sample space.

Lesson 7.2.1: Introducing Conditional Probability

Introduction

Let's say you and your friends draw straws to see who has to do some unpleasant activity, like cleaning out the class pet's cage. If everyone who draws before you keeps their straw, does that affect your odds of cleaning up after Fluffy the Hamster?

If you are drawing straws without replacing them, your friends' outcomes do have an effect on yours—if there are several long straws and one unfortunate short one, your odds of drawing the short straw increase with every long straw drawn.

In this lesson, we will look at conditional probability—that is, the probability that an event will occur based on the condition that another event has occurred.

Key Concepts

- The **conditional probability of *B* given *A*** is the probability that event *B* occurs, given that event *A* has already occurred.
- If *A* and *B* are two events from a sample space with $P(A) \neq 0$, then the conditional probability of *B* given *A*, denoted $P(B|A)$, has two equivalent expressions:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\text{number of outcomes in } (A \text{ and } B)}{\text{number of outcomes in } A}$$

- Here is an explanation of these equivalent expressions:

$P(B\|A) = \frac{\text{number of outcomes in } (A \text{ and } B)}{\text{number of outcomes in } A}$	This uses subsets of the sample space.
$P(B\|A) = \frac{P(A \text{ and } B)}{P(A)}$	This uses the calculated probabilities *P*(*A* and *B*) and *P*(*A*).

- The second formula can be rewritten as $P(A \text{ and } B) = P(A) \bullet P(B|A)$.
- $P(B|A)$ is read "the probability of *B* given *A*."
- Using set notation, conditional probability is written like this:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- The "conditional probability of *B* given *A*" only has meaning if event *A* has occurred. That is why the formula for $P(B|A)$ has the requirement that $P(A) \neq 0$.

- The conditional probability formula can be solved to obtain a formula for *P*(*A* and *B*):

$P(B \vert A) = \dfrac{P(A \text{ and } B)}{P(A)}$	Write the conditional probability formula.
$P(A) \bullet P(B \vert A) = \dfrac{P(A \text{ and } B)}{P(A)} \bullet P(A)$	Multiply both sides by *P*(*A*).
$P(A) \bullet P(B \vert A) = P(A \text{ and } B)$	Simplify.
$P(A \text{ and } B) = P(A) \bullet P(B \vert A)$	Reverse the left and right sides.

- Remember that independent events are two events such that the probability of both events occurring is equal to the product of the individual probabilities. Two events *A* and *B* are independent if and only if *P*(*A* and *B*) = *P*(*A*) • *P*(*B*). Using set notation, $P(A \cap B) = P(A) \bullet P(B)$. The occurrence or non-occurrence of one event has no effect on the probability of the other event.

- If *A* and *B* are independent, then the formula for *P*(*A* and *B*) is the equation used in the definition of independent events, as shown:

$P(A \text{ and } B) = P(A) \bullet P(B \vert A)$	formula for *P*(*A* and *B*)
$P(A \text{ and } B) = P(A) \bullet P(B)$	formula for *P*(*A* and *B*) if *A* and *B* are independent

- Using the conditional probability formula in different situations requires using different variables, depending on how the events are named. Here are a couple examples.

$P(B \vert A) = \dfrac{P(A \text{ and } B)}{P(A)}$	Use this equation to find the probability of *B* given *A*.
$P(A \vert B) = \dfrac{P(A \text{ and } B)}{P(B)}$	Use this equation to find the probability of *A* given *B*.

- Another method to use when calculating conditional probabilities is dividing the number of outcomes in the intersection of *A* and *B* by the number of outcomes in a certain event:

$$P(A \vert B) = \frac{\text{number of outcomes in } A \cap B}{\text{number of outcomes in } B}$$

- The following statements are equivalent. In other words, if any one of them is true, then the others are all true.
  - Events *A* and *B* are independent.
  - The occurrence of *A* has no effect on the probability of *B*; that is, $P(B|A)=P(B)$.
  - The occurrence of *B* has no effect on the probability of *A*; that is, $P(A|B)=P(A)$.
  - $P(A \text{ and } B) = P(A) \bullet P(B)$
- *Note*: For real-world data, these modified tests for independence are sometimes used:
  - Events *A* and *B* are independent if the occurrence of *A* has no *significant* effect on the probability of *B*; that is, $P(B|A)\approx P(B)$.
  - Events *A* and *B* are independent if the occurrence of *B* has no *significant* effect on the probability of *A*; that is, $P(A|B)\approx P(A)$.
- When using these modified tests, good judgment must be used when deciding whether the probabilities are close enough to conclude that the events are independent.

Guided Practice 7.2.1

Example 1

Alexis rolls a pair of number cubes. What is the probability that both numbers are odd if their sum is 6? Interpret your answer in terms of a uniform probability model.

1. Assign variable names to the events and state what you need to find, using conditional probability.

   Let A be the event "Both numbers are odd."

   Let B be the event "The sum of the numbers is 6."

   You need to find the probability of A given B.

   That is, you need to find $P(A|B)$.

2. Show the sample space.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

   Key: (2, 3) means 2 on the first cube and 3 on the second cube.

3. Identify the outcomes in the events.

The outcomes for each sample space are shaded.

***A*: Both numbers are odd.**

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

***B*: The sum of the numbers is 6.**

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

4. Identify the outcomes in the events $A \cap B$ and B.

Use the conditional probability formula: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

$A \cap B$ = the outcomes that are in A and also in B.

$A \cap B = \{(1, 5), (3, 3), (5, 1)\}$

$B = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

5. Find $P(A \cap B)$ and $P(B)$.

$A \cap B$ has 3 outcomes; the sample space has 36 outcomes.

$$P(A \cap B) = \frac{\text{number of outcomes in } A \text{ and } B}{\text{number of outcomes in sample space}} = \frac{3}{36}$$

B has 5 outcomes; the sample space has 36 outcomes.

$$P(B) = \frac{\text{number of outcomes in } B}{\text{number of outcomes in sample space}} = \frac{5}{36}$$

6. Find $P(A|B)$.

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$ Write the conditional probability formula.

$$P(A|B)=\frac{\frac{3}{36}}{\frac{5}{36}}$$ Substitute the probabilities found in step 5.

$$P(A|B)=\frac{3}{36}\bullet\frac{36}{5}$$ To divide by a fraction, multiply by its reciprocal.

$$P(A|B)=\frac{3}{{}_{1}\cancel{36}}\bullet\frac{\cancel{36}^{1}}{5}$$ Simplify.

$$P(A|B)=\frac{3}{5}$$

7. Verify your answer.

Use this alternate conditional probability formula:

$$P(A|B)=\frac{\text{number of outcomes in } A\cap B}{\text{number of outcomes in } B}.$$

$A\cap B$ has 3 outcomes: {(1, 5), (3, 3), (5, 1)}.

B has 5 outcomes: {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}.

$$P(A|B)=\frac{\text{number of outcomes in } A\cap B}{\text{number of outcomes in } B}$$

$$P(A|B)=\frac{3}{5}$$

8. Interpret your answer in terms of a uniform probability model.

The probabilities used in solving the problem are found by using the ratios $\frac{\text{number of outcomes in an event}}{\text{number of outcomes in the sample space}}$ and $\frac{\text{number of outcomes in an event}}{\text{number of outcomes in a subset of the sample space}}$.

These ratios are uniform probability models if all outcomes in the sample space are equally likely. It is reasonable to assume that Alexis rolls fair number cubes, so all outcomes in the sample space are equally likely. Therefore, the answer is valid and can serve as a reasonable predictor. You can predict the following: If you roll a pair of number cubes a large number of times and consider all the outcomes that have a sum of 6, then about $\frac{3}{5}$ of those outcomes will have both odd numbers.

Example 2

Hamid rolls a number cube 3 times. Consider the following events.

A: The first roll is an odd number.

B: There are exactly 2 consecutive odd numbers.

Determine if *A* and *B* are independent events.

1. Investigate the sample space.

   The number cube is rolled 3 times, so the outcomes are ordered triples.

(1, 1, 1)	(1, 1, 2)	(1, 1, 3)	(1, 1, 4)	(1, 1, 5)	(1, 1, 6)
(1, 2, 1)	(1, 2, 2)	(1, 2, 3)	(1, 2, 4)	(1, 2, 5)	(1, 2, 6)
(1, 3, 1)	(1, 3, 2)	(1, 3, 3)	(1, 3, 4)	(1, 3, 5)	(1, 3, 6)
(1, 4, 1)	(1, 4, 2)	(1, 4, 3)	(1, 4, 4)	(1, 4, 5)	(1, 4, 6)
(1, 5, 1)	(1, 5, 2)	(1, 5, 3)	(1, 5, 4)	(1, 5, 5)	(1, 5, 6)
(1, 6, 1)	(1, 6, 2)	(1, 6, 3)	(1, 6, 4)	(1, 6, 5)	(1, 6, 6)
(2, 1, 1)	(2, 1, 2)	(2, 1, 3)	(2, 1, 4)	(2, 1, 5)	(2, 1, 6)
(2, 2, 1)	(2, 2, 2)	(2, 2, 3)	(2, 2, 4)	(2, 2, 5)	(2, 2, 6)
(2, 3, 1)	(2, 3, 2)	etc.			

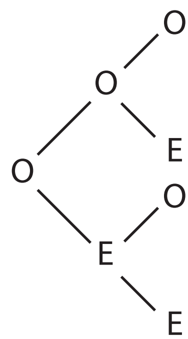
   The ordered triples listed above comprise only a small part of the sample space, which contains $6 \cdot 6 \cdot 6 = 216$ outcomes.

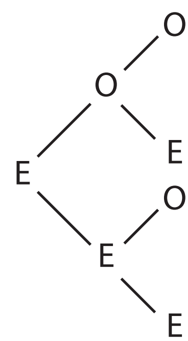
   See step 2 for a more efficient method of determining the sample space.

2. Create a tree diagram for the outcomes.

For the events under consideration, you need only know whether each number cube roll is odd or even. The possible results of a number cube roll are 1, 2, 3, 4, 5, and 6. Half of the numbers on the number cube are odd and half of them are even, so odd and even are equally likely. Therefore, you can consider a sample space comprised of the outcomes "odd" and "even."

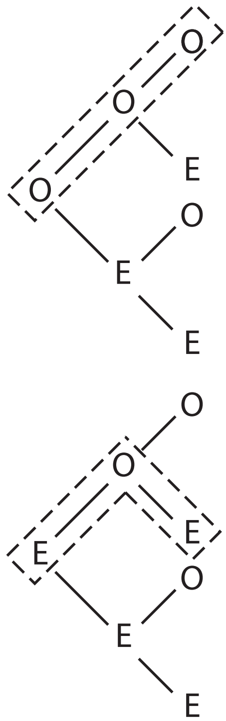
The tree diagram below shows all possible outcomes, using O for odd and E for even.





To identify all the possible outcomes of 3 number cube rolls, trace all the different paths from left to right. For example, the diagram that follows shows two different paths.

(*continued*)



There are eight different paths that can be found from this new sample space.

Sample space = {OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

3. Events A and B are independent if $P(A|B) = P(A)$ and $P(B|A) = P(B)$. Find the probabilities of the relevant events.

A: The first roll is odd.

{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

B: There are exactly 2 consecutive odds.

{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$$P(B) = \frac{2}{8} = \frac{1}{4}$$

(continued)

$A|B$: The first roll is odd, given that there are exactly 2 consecutive odds.

{OOE, EOO}

$$P(A|B)=\frac{1}{2}$$

$B|A$: There are exactly 2 consecutive odds, given that the first roll is odd.

{OOO, OOE, OEO, OEE}

$$P(B|A)=\frac{1}{4}$$

4. Determine if A and B are independent.

A and B are independent if $P(A|B)=P(A)$ and $P(B|A)=P(B)$.

First, determine if $P(A|B)=P(A)$.

$P(A|B)=P(A)$ Set $P(A|B)$ equal to $P(A)$.

$\frac{1}{2}=\frac{1}{2}$ Substitute probabilities for $P(A|B)$ and $P(A)$.

This verifies that $P(A|B)=P(A)$.

Now, determine if $P(B|A)=P(B)$.

$P(B|A)=P(B)$ Set $P(B|A)$ equal to $P(B)$.

$\frac{1}{4}=\frac{1}{4}$ Substitute probabilities for $P(B|A)$ and $P(B)$.

This verifies that $P(B|A)=P(B)$.

A and B are independent.

5. Check your results.

The definition of independent events contains a different test for independence: *A* and *B* are independent if $P(A \cap B) = P(A) \bullet P(B)$. Use this other test to verify your answer.

A: The first roll is odd.

{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

B: There are exactly 2 consecutive odds.

{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$$P(B) = \frac{2}{8} = \frac{1}{4}$$

$A \cap B$: The first roll is odd and there are exactly 2 consecutive odds.

{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE}

$$P(A \cap B) = \frac{1}{8}$$

Verify.

$P(A \cap B) = P(A) \bullet P(B)$	Set $P(A \cap B)$ equal to $P(A) \bullet P(B)$.
$\frac{1}{8} = \frac{1}{2} \bullet \frac{1}{4}$	Substitute probabilities for $P(A \cap B)$, $P(A)$, and $P(B)$.
$\frac{1}{8} = \frac{1}{8}$	Simplify.

A and *B* are independent.

Example 3

A vacation resort offers bicycles and personal watercrafts for rent. The resort's manager made the following notes about rentals:

- 200 customers rented items in all—100 rented bicycles and 100 rented personal watercrafts.
- Of the personal watercraft customers, 75 customers were young (30 years old or younger) and 25 customers were older (31 years old or older).
- 125 of the 200 customers were age 30 or younger. 50 of these customers rented bicycles, and 75 of them rented personal watercrafts.

Consider the following events that apply to a random customer.

Y: The customer is young (30 years old or younger).

W: The customer rents a personal watercraft.

Are Y and W independent? Compare $P(Y|W)$ and $P(W|Y)$ and interpret the results.

1. Determine if Y and W are independent.

First, determine the probabilities of each event.

$P(Y)=\frac{125}{200}=0.625$	Of 200 customers, 125 were young.
$P(W)=\frac{100}{200}=0.5$	Of 200 customers, 100 rented a personal watercraft.
$P(Y\|W)=\frac{75}{100}=0.75$	Of 100 personal watercraft customers, 75 were young.
$P(W\|Y)=\frac{75}{125}=0.6$	Of 125 young customers, 75 rented a personal watercraft.

Y and W are dependent because $P(Y|W)\neq P(Y)$ and $P(W|Y)\neq P(W)$.

2. Compare $P(Y|W)$ and $P(W|Y)$.

$P(Y|W)=0.75$

$P(W|Y)=0.6$

0.75 > 0.6; therefore, $P(Y|W)>P(W|Y)$.

3. Interpret the results.

$P(Y|W)$ represents the probability that a customer is young given that the customer rents a personal watercraft.

$P(W|Y)$ represents the probability that a customer rents a personal watercraft given that the customer is young.

The dependence of the events Y and W means that a customer's age affects the probability that the customer rents a personal watercraft; in this case, being young increases that probability because $P(W|Y)>P(W)$. The dependence of the events Y and W also means that a customer renting a personal watercraft affects the probability that the customer is young; in this case, renting a personal watercraft increases that probability because $P(Y|W)>P(Y)$.

$P(Y|W)>P(W|Y)$ means that it is more likely that a customer is young given that he or she rents a personal watercraft than it is that a customer rents a personal watercraft given that he or she is young.

Example 4

Last year at All Technical High School, 20% of the students received an academic award. That same year, 16% of the students at the school received an academic award and a service award. What is the probability that a student who receives an academic award also receives a service award?

1. Restate the question using a conditional probability.

   The original question asked, "What is the probability that a student who receives an academic award also receives a service award?"

   Rephrased in terms of a conditional probability, the question can be posed, "What is the probability that a student receives a service award given that he receives an academic award?"

2. Assign variable names to the relevant events and state what you need to find.

   Let S be the event "A student receives a service award."

   Let A be the event "A student receives an academic award."

   Then $S \cap A$ is the event "A student receives a service award and an academic award."

   Find $P(S|A)$.

3. Write the probabilities you know, based on the data given in the problem.

   $P(A) = 0.20$

   $P(S \cap A) = 0.16$

4. Apply the conditional probability formula.

$P(S\mid A)=\dfrac{P(S\cap A)}{P(A)}$	Write the conditional probability formula for S and A.
$P(S\mid A)=\dfrac{0.16}{0.20}$	Substitute the probabilities determined in step 3.
$P(S\mid A)=0.8$	Simplify.

There is an 80% probability that a student who receives an academic award also receives a service award.

Practice 7.2.1: Introducing Conditional Probability

Denise rolls a pair of number cubes. Use this information to complete problems 1 and 2.

1. What is the probability that both numbers are even if their sum is 8? What is the probability that the sum of the numbers is 8 if both numbers are even?

2. What is the probability that the sum is odd if the product is odd? What is the probability that the product is odd if the sum is odd?

For problems 3 and 4, list the sample space for the given events and explain what your symbols represent. Use conditional probability to determine if events *A* and *B* are independent; show your work.

3. Duane rolls a number cube 3 times.

   Event *A*: The first roll is greater than 3.

   Event *B*: The third roll is less than 4.

4. The Basses have 3 children.

   Event *A*: The oldest is a boy.

   Event *B*: There are exactly 2 boys.

continued

Use the following information to complete problems 5 and 6.

A surveyor collected data from 161 students regarding their interests and favorite entertainment activities. When asked to choose either "music and drama" or "athletics" as their greater overall interest, 80 students chose music and drama, while 81 students chose athletics. Students were then asked to choose among several entertainment activities, including going to concerts or going bowling.

- 41 students chose concerts; of those students, 20 had chosen music and drama, while 21 had chosen athletics.
- 40 students chose bowling; of those students, 10 had chosen music and drama, while 30 had chosen athletics.

5. Consider the following events.

   MD: A student chooses music and drama.

   C: A student chooses concerts.

   Are *MD* and *C* independent? Compare $P(MD|C)$ and $P(C|MD)$. Interpret what your answers mean.

6. Consider the following events.

   A: A student chooses athletics.

   B: A student chooses bowling.

   Are *A* and *B* independent? Compare $P(A|B)$ and $P(B|A)$. Interpret what your answers mean.

continued

Lesson 2: Conditional Probability

Read each scenario and use the information to answer questions 7–10.

7. A total of 20% of a fire department's firefighters are in the Special Units Division. A total of 8% of the department's firefighters are in the Special Units Division and have a college degree. What is the probability that a firefighter in the Special Units Division has a college degree?

8. The coach of the Strikers soccer team has determined that 50% of the players on the team were also on the team last year. The coach also knows that 30% of the players on the team were on the team last year and attended soccer camp during the summer. What is the probability that a Striker who was on the team last year attended soccer camp during the summer?

9. Andre rolls a pair of number cubes. What is the probability that the sum is 7 if the numbers are different? What is the probability that the numbers are different if the sum is 7?

10. Liliana tosses a coin 3 times. What is the probability that she gets exactly 2 heads if the second toss is heads? What is the probability that the second toss is heads if she gets exactly 2 heads?

Lesson 7.2.2: Using Two-Way Frequency Tables

Introduction

Data can be presented in many different ways. In the previous lesson, you worked with data in conditional probabilities to interpret the relationships between various events. When you are presented with a great deal of data, two-way frequency tables can provide a convenient way to compare the frequency of data items and see the relationships among them.

Key Concepts

- Previously, you learned that there are two equivalent expressions for the conditional probability of *B* given *A*:

$$P(B|A)=\frac{P(A \text{ and } B)}{P(A)}=\frac{\text{number of outcomes in }(A \text{ and } B)}{\text{number of outcomes in } A}$$

- Conditional probability can be used to test for independence.
- Events *A* and *B* are independent events if $P(B|A)=P(B)$ or if $P(A|B)=P(A)$. (Note that if one is true, then the other is also true.)
- Remember that for real-world data, modified tests for independence are sometimes used:
    - Events *A* and *B* are independent if the occurrence of *A* has no *significant* effect on the probability of *B*; that is, $P(B|A)\approx P(B)$.
    - Events *A* and *B* are independent if the occurrence of *B* has no *significant* effect on the probability of *A*; that is, $P(A|B)\approx P(A)$.
- If *A* and *B* are two events from a sample space with $P(A) \neq 0$, then the conditional probability of *B* given *A* in set notation is $P(B|A)=\frac{P(A\cap B)}{P(A)}$.
- The conditional probability formula can be solved to obtain this formula for *P*(*A* and *B*):

$$P(A \text{ and } B)=P(A)\bullet P(B|A)$$

- A **two-way frequency table** is a frequency table that shows two categories of characteristics, one in rows and the other in columns. Each cell value is a frequency that shows how many times two different characteristics appear together, or how often characteristics are associated with a person, object, or type of item that is being studied.
- The following example shows a typical setup of a two-way frequency table.

Category 1 of characteristics	**Category 2 of characteristics**	
	Characteristic 1	**Characteristic 2**
Characteristic 1	a	b
Characteristic 2	c	d

- When probabilities and conditional probabilities are calculated from data in a two-way frequency table, then totals of characteristics are needed.

Category 1 of characteristics	**Category 2 of characteristics**		**Total**
	Characteristic 1	**Characteristic 2**	
Characteristic 1	a	b	
Characteristic 2	c	d	
Total			

- When a two-way frequency table is used to find probabilities and conditional probabilities, the characteristics represent events, and the frequencies are numbers of outcomes.

Guided Practice 7.2.2

Example 1

The Student Council wants to host a school-wide activity. Council members survey 40 students, asking them to choose either a field trip, a dance, or a talent show. The table below shows the survey results, with the surveyed students numbered 1–40. Construct a two-way frequency table to summarize the data.

Student	Grade	Activity	Student	Grade	Activity
1	10	FT	21	10	D
2	12	D	22	10	FT
3	10	TS	23	12	D
4	10	FT	24	11	D
5	11	D	25	11	TS
6	12	D	26	12	D
7	10	TS	27	12	D
8	10	FT	28	10	D
9	10	FT	29	11	D
10	11	TS	30	11	D
11	12	D	31	12	FT
12	10	TS	32	10	TS
13	11	TS	33	12	D
14	10	FT	34	11	D
15	11	D	35	11	FT
16	10	FT	36	11	FT
17	12	D	37	11	TS
18	10	FT	38	12	TS
19	12	D	39	11	FT
20	11	TS	40	12	TS

Key: TS = Talent show, FT = Field trip, D = Dance

1. Set up a tally table.

There are two characteristics associated with each student: that student's grade and that student's choice of activity. Set up a table that shows "Grade" and "Activity choice" as categories, and all the different characteristics in each category.

Grade	**Activity choice**		
	Talent show	**Field trip**	**Dance**
10			
11			
12			

2. Tally the data.

For each student, draw a tally mark that corresponds to that student's grade and choice of activity in the appropriate cell of the data table. The tally marks for students 1–5 are shown in the incomplete tally table below.

Grade	**Activity choice**		
	Talent show	**Field trip**	**Dance**
10	I	II	
11			I
12			I

The complete tally table below shows the tally marks for all the students.

Grade	**Activity choice**		
	Talent show	**Field trip**	**Dance**
10	IIII	~~IIII~~ III	II
11	~~IIII~~	III	~~IIII~~ I
12	II	I	~~IIII~~ IIII

3. Create a two-way frequency table.

Count the tally marks in each cell of your tally table. Then, create another table (a two-way frequency table) to show your count results. These results are frequencies. The completed two-way frequency table is shown below.

Grade	Activity choice		
	Talent show	Field trip	Dance
10	4	8	2
11	5	3	6
12	2	1	9

Add all the frequencies; verify that their sum is 40 (since 40 students were surveyed).

$4 + 8 + 2 + 5 + 3 + 6 + 2 + 1 + 9 = 40$

✓

Example 2

The completed two-way frequency table from Example 1 is shown below. It shows the results of a survey designed to help the Student Council choose a school-wide activity.

Grade	Activity choice		
	Talent show	Field trip	Dance
10	4	8	2
11	5	3	6
12	2	1	9

Consider the following events that apply to a random student who participated in the survey.

TEN: The student is in the tenth grade.

TWELVE: The student is in the twelfth grade.

FT: The student prefers a field trip.

TS: The student prefers a talent show.

Compare $P(TEN|FT)$ and $P(FT|TEN)$. Are *TEN* and *FT* independent?

Compare $P(TWELVE|TS)$ and $P(TS|TWELVE)$. Are *TWELVE* and *TS* independent?

Interpret the results.

1. Find the totals of all the categories.

Grade	Activity choice			Total
	Talent show	Field trip	Dance	
10	4	8	2	14
11	5	3	6	14
12	2	1	9	12
Total	11	12	17	40

2. Compare $P(TEN|FT)$ and $P(FT|TEN)$.

$P(TEN|FT)=\frac{8}{12}\approx 0.667$ There were 12 votes for a field trip; 8 were by tenth graders.

$P(FT|TEN)=\frac{8}{14}\approx 0.571$ There were 14 votes by tenth graders; 8 were for a field trip.

0.667 > 0.571; therefore, $P(TEN|FT)>P(FT|TEN)$.

3. Determine if *TEN* and *FT* are independent.

Remember that events *A* and *B* are independent events if $P(B|A) = P(B)$ or if $P(A|B) = P(A)$.

Compare $P(TEN|FT)$ with $P(TEN)$ and $P(FT|TEN)$ with $P(FT)$.

$P(TEN|FT) = \frac{8}{12} \approx 0.667$ — There were 12 votes for a field trip; 8 were by tenth graders.

$P(TEN) = \frac{14}{40} = 0.35$ — There were 40 votes in all; 14 were by tenth graders.

$0.667 \neq 0.35$; therefore, $P(TEN|FT) \neq P(TEN)$.

$P(FT|TEN) = \frac{8}{14} \approx 0.571$ — There were 14 votes by tenth graders; 8 were for a field trip.

$P(FT) = \frac{12}{40} = 0.3$ — There were 40 votes in all; 12 were for a field trip.

$0.571 \neq 0.3$; therefore, $P(FT|TEN) \neq P(FT)$.

Based on the data, *TEN* and *FT* seem to be dependent because $P(TEN|FT) \neq P(TEN)$ and $P(FT|TEN) \neq P(FT)$.

4. Interpret the results for $P(TEN|FT)$ and $P(FT|TEN)$.

$P(TEN|FT)$ is the probability that a student is in the tenth grade given that he prefers a field trip.

$P(FT|TEN)$ is the probability that a student prefers a field trip given that he is in the tenth grade.

The fact that *TEN* and *FT* are dependent means that being in the tenth grade affects the probability that a student prefers a field trip, and preferring a field trip affects the probability that a student is in the tenth grade. In this case, being in the tenth grade increases the probability that a student prefers a field trip because $P(FT|TEN) > P(FT)$. Also, preferring a field trip increases the probability that a student is in the tenth grade because $P(TEN|FT) > P(TEN)$.

$P(TEN|FT) > P(FT|TEN)$ means that it is more likely that a student is in the tenth grade given that he prefers a field trip than it is that a student prefers a field trip given that he is in the tenth grade.

5. Compare $P(TWELVE|TS)$ and $P(TS|TWELVE)$.

$P(TWELVE|TS) = \frac{2}{11} \approx 0.182$ There were 11 votes for a talent show; 2 were by twelfth graders.

$P(TS|TWELVE) = \frac{2}{12} \approx 0.167$ There were 12 votes by twelfth graders; 2 were for a talent show.

$0.182 > 0.167$; therefore, $P(TWELVE|TS) > P(TS|TWELVE)$, but they are close in value. The values are approximately 18% and 17%.

6. Determine if *TWELVE* and *TS* are independent.

Events *A* and *B* are independent events if $P(B|A) = P(B)$ or if $P(A|B) = P(A)$.

Compare $P(TWELVE|TS)$ with $P(TWELVE)$ and $P(TS|TWELVE)$ with $P(TS)$.

$P(TWELVE|TS) = \frac{2}{11} \approx 0.182$ There were 11 votes for a talent show; 2 were by twelfth graders.

$P(TWELVE) = \frac{12}{40} = 0.3$ There were 40 votes in all; 12 were by twelfth graders.

$0.182 \neq 0.3$; therefore, $P(TWELVE|TS) \neq P(TWELVE)$.

$P(TS|TWELVE) = \frac{2}{12} \approx 0.167$ There were 12 votes by twelfth graders; 2 were for a talent show.

$P(TS) = \frac{11}{40} = 0.275$ There were 40 votes in all; 11 were for a talent show.

$0.167 \neq 0.275$; therefore, $P(TS|TWELVE) \neq P(TS)$.

Based on the data, *TWELVE* and *TS* are dependent because $P(TWELVE|TS) \neq P(TWELVE)$ and $P(TS|TWELVE) \neq P(TS)$.

7. Interpret the results for $P(TWELVE|TS)$ and $P(TS|TWELVE)$.

$P(TWELVE|TS)$ is the probability that a student is in the twelfth grade given that the student prefers a talent show.

$P(TS|TWELVE)$ is the probability that a student prefers a talent show given that the student is in the twelfth grade.

The fact that *TWELVE* and *TS* are dependent means that being in the twelfth grade affects the probability that a student prefers a talent show, and preferring a talent show affects the probability that a student is in the twelfth grade. In this case, being in the twelfth grade decreases the probability that a student prefers a talent show because $P(TS|TWELVE) < P(TS)$. And preferring a talent show decreases the probability that a student is in the twelfth grade because $P(TWELVE|TS) < P(TWELVE)$.

$P(TWELVE|TS) > P(TS|TWELVE)$, but they are close in value, differing by only about 1%. So it is almost equally likely that a student is in the twelfth grade given that the student prefers a talent show, as it is that a student prefers a talent show given that the student is in the twelfth grade.

Example 3

A cafeteria manager recorded the choices of 100 students who each chose one food item and one beverage. The table shows the data.

Beverage choice	Food choice	
	Burger	Pizza
Milk	34	26
Iced tea	24	16

Consider the following events that apply to a randomly chosen student who selects one food item and one beverage.

M: The student selects milk.

B: The student selects a burger.

P: The student selects pizza.

Compare $P(M|B)$ and $P(B|M)$. Are M and B independent?

Compare $P(M|P)$ and $P(P|M)$. Are M and P independent?

Interpret the results.

1. Find the totals of all the categories.

Beverage choice	Food choice		Total
	Burger	Pizza	
Milk	34	26	60
Iced tea	24	16	40
Total	58	42	100

2. Compare $P(M|B)$ and $P(B|M)$.

$P(M|B)=\frac{34}{58}\approx 0.586$ — 58 burgers were chosen; 34 were accompanied by milk.

$P(B|M)=\frac{34}{60}\approx 0.567$ — 60 servings of milk were chosen; 34 were accompanied by a burger.

$0.586 > 0.567$; therefore, $P(M|B) > P(B|M)$, but they are close in value. The values are approximately 59% and 57%.

3. Determine if M and B are independent.

Events A and B are independent if $P(B|A)=P(B)$ or if $P(A|B)=P(A)$.

Compare $P(M|B)$ with $P(M)$ and $P(B|M)$ with $P(B)$.

$P(M|B)=\frac{34}{58}\approx 0.586$ — 58 burgers were chosen; 34 were accompanied by milk.

$P(M)=\frac{60}{100}=0.6$ — There were 100 students, and 60 of them chose milk.

$0.586 \approx 0.6$; therefore, $P(M|B)\approx P(M)$.

$P(B|M)=\frac{34}{60}\approx 0.567$ — 60 servings of milk were chosen; 34 were accompanied by a burger.

$P(B)=\frac{58}{100}=0.58$ — There were 100 students, and 58 of them chose a burger.

$0.567 \approx 0.58$; therefore, $P(B|M)\approx P(B)$.

Based on the data, M and B seem to be independent because $P(M|B)\approx P(M)$ and $P(B|M)\approx P(B)$.

4. Interpret the results for $P(M|B)$ and $P(B|M)$.

$P(M|B)$ represents the probability of a student choosing milk given that he chooses a burger.

$P(B|M)$ represents the probability of a student choosing a burger given that he chooses milk.

The conclusion that M and B are independent based on probabilities that are close in value means that choosing milk does not significantly affect the probability of choosing a burger, and choosing a burger does not significantly affect the probability of choosing milk.

$P(M|B) > P(B|M)$, but they are close in value, differing by only about 1%. So it is almost equally likely that a student chooses milk given that the student chooses a burger, as it is that the student chooses a burger given that the student chooses milk. But because the choices are independent based on probabilities that are close in value, this can be restated more simply as follows: It is almost equally likely that a student chooses milk as that the student chooses a burger.

5. Compare $P(M|P)$ and $P(P|M)$.

$$P(M|P)=\frac{26}{42}\approx 0.619$$

42 pizza servings were chosen; 26 were accompanied by milk.

$$P(P|M)=\frac{26}{60}\approx 0.433$$

60 servings of milk were chosen; 26 were accompanied by pizza.

$0.619 > 0.433$; therefore, $P(M|P) > P(P|M)$.

6. Determine if M and P are independent.

Events A and B are independent events if $P(B|A) = P(B)$ or if $P(A|B) = P(A)$.

Compare $P(M|P)$ with $P(M)$ and $P(P|M)$ with $P(P)$.

$P(M\|P)=\dfrac{26}{42}\approx 0.619$	42 pizza servings were chosen; 26 were accompanied by milk.
$P(M)=\dfrac{60}{100}=0.6$	There were 100 students, and 60 of them chose milk.

$0.619 \approx 0.6$; therefore, $P(M|P) \approx P(M)$.

$P(P\|M)=\dfrac{26}{60}\approx 0.433$	60 servings of milk were chosen; 26 were accompanied by pizza.
$P(P)=\dfrac{42}{100}=0.42$	There were 100 students, and 42 of them chose pizza.

$0.433 \approx 0.42$; therefore, $P(P|M) \approx P(P)$.

Based on the data, M and P seem to be independent because $P(M|P) \approx P(M)$ and $P(P|M) \approx P(P)$.

7. Interpret the results for $P(M|P)$ and $P(P|M)$.

$P(M|P)$ represents the probability that a student chooses milk given that the student chooses pizza.

$P(P|M)$ represents the probability that a student chooses pizza given that the student chooses milk.

The conclusion that M and P are independent based on probabilities that are close in value means that choosing milk does not significantly affect the probability of choosing pizza, and choosing pizza does not significantly affect the probability of choosing milk.

$P(M|P) > P(P|M)$ means it is more likely that a student chooses milk given that the student chooses pizza than it is that a student chooses pizza given that the student chooses milk. But since the choices are independent based on probabilities that are close in value, this can be restated as follows: It is more likely that a student chooses milk than that a student chooses pizza.

Practice 7.2.2: Using Two-Way Frequency Tables

A survey was conducted of 20 students who watch sports on television. They were asked to choose which sport they most like watching: football, basketball, baseball, or softball. The table shows the students' answers along with their genders. The students are numbered 1–20. Use the table to complete problems 1 and 2.

Student	Sport	Gender	Student	Sport	Gender
1	Football	M	11	Basketball	M
2	Baseball	F	12	Softball	M
3	Basketball	F	13	Basketball	F
4	Football	M	14	Football	M
5	Basketball	M	15	Softball	F
6	Football	M	16	Football	M
7	Football	F	17	Softball	F
8	Baseball	M	18	Basketball	F
9	Softball	F	19	Football	M
10	Football	F	20	Football	M

Key: M = Male, F = Female

1. Set up and complete a tally table for the data.

2. Use your tally table to construct a two-way frequency table that summarizes the data.

continued

UNIT 7 • APPLICATIONS OF PROBABILITY
Lesson 2: Conditional Probability

Every student in Western High School's work-study program has either a paid job or a volunteer position in the community. The table shows the data by grade. Use the table and the events that follow to complete problems 3–6.

Grade	**Work category**	
	Paid job	**Volunteer position**
10	18	6
11	24	53
12	65	14

TEN: A work-study student is in the tenth grade.

TWELVE: A work-study student is in the twelfth grade.

P: A work-study student has a paid job.

V: A work-study student has a volunteer position.

3. Compare $P(TEN|V)$ and $P(V|TEN)$. Explain what your answer means.

4. Are *TEN* and *V* independent? Explain your reasoning and what your answer means.

5. Compare $P(TWELVE|P)$ and $P(P|TWELVE)$. Explain what your answer means.

6. Are *TWELVE* and *P* independent? Explain your reasoning and what your answer means.

continued

Surveyors asked 160 students to choose which activity they most like doing with friends: talking on the phone, playing video games, listening to music, or hanging out at the mall. The table shows the students' answers along with their genders. Use the table and the events that follow to complete problems 7–10.

Gender	Preferred activity			
	Phone	Video games	Music	Mall
Male	12	25	20	20
Female	19	15	21	18

M: The student is male.

F: The student is female.

P: The student prefers talking on the phone.

V: The student prefers playing video games.

MU: The student prefers listening to music.

MA: The student prefers hanging out at the mall.

7. Compare $P(M|P)$ and $P(P|M)$. Determine if *M* and *P* are independent. Show the numerical values of all the probabilities used in your answers.

8. Compare $P(F|V)$ and $P(V|F)$. Determine if *F* and *V* are independent. Show the numerical values of all the probabilities used in your answers.

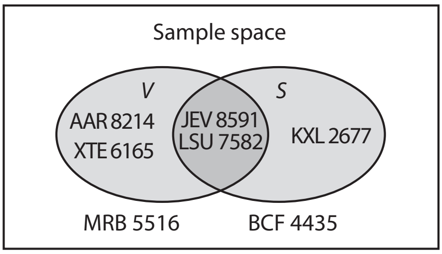
9. Compare $P(M|MU)$ and $P(MU|M)$. Determine if *M* and *MU* are independent. Show the numerical values of all the probabilities used in your answers.

10. Compare $P(F|MA)$ and $P(MA|F)$. Determine if *F* and *MA* are independent. Show the numerical values of all the probabilities used in your answers.

Answer Key

Lesson 1: Events

Practice 7.1.1: Describing Events, pp. 21–23

1. The chosen house has a fireplace and oil heat.
3. {Opal, Colin, Hugo, Carla, Rick}
5. 

7. *C*
9. 10

Practice 7.1.2: The Addition Rule, pp. 31–33

1. 3/4 = 0.75 = 75%
3. 5/24
5. 1/6 ≈ 0.17 = 17%
7. 0
9. 0

Practice 7.1.3: Understanding Independent Events, pp. 44–46

1. a. no
   b. yes
   c. no
   d. yes
3. 1/10, or 10%; dependent
5. Dependent. Supporting work: *P*(Jones and less than high school) = 2/114 ≈ 0.017; *P*(Jones) • *P*(less than high school) = (35/114) • (12/114) ≈ 0.032. Therefore, *P*(Jones and less than high school) ≠ *P*(Jones) • *P*(less than high school).
7. 0.4 or 40%
9. 0.75 or 75%

Lesson 2: Conditional Probability

Practice 7.2.1: Introducing Conditional Probability, pp. 65–67

1. *P*(both numbers are even | sum = 8) = 3/5; *P*(sum = 8 | both numbers are even) = 3/9 = 1/3
3. Sample space = {GGG, GGL, GLG, GLL, LGG, LGL, LLG, LLL}, where G represents greater than 3 on the first roll and L represents less than 4 on the third roll; *A* and *B* are independent. Explanation: $P(A|B) = P(A)$ and $P(B|A) = P(B)$; Probabilities: $P(A) = 4/8 = 1/2$; $P(A|B) = 2/4 = 1/2$; $P(B) = 4/8 = 1/2$; $P(B|A) = 2/4 = 1/2$

5. MD and C are independent because $P(C \mid MD) \approx P(C)$ and $P(MD \mid C) \approx P(MD)$; $P(C \mid MD) = 20/80 = 0.25$; $P(C) = 41/161 \approx 0.255$; $P(MD \mid C) = 20/41 \approx 0.488$; $P(MD) = 80/161 \approx 0.497$. Compare $P(MD \mid C)$ and $P(C \mid MD)$: $P(MD \mid C) > P(C \mid MD)$.
   Interpretation: $P(C \mid MD)$ represents the probability that a student chooses a concert given that she chooses music and drama. $P(MD \mid C)$ represents the probability that a student chooses music and drama given that she chooses a concert. The independence of the events C and MD means that choosing a concert does not affect the probability of choosing music and drama, and vice versa. $P(MD \mid C) > P(C \mid MD)$ means that it is more likely that a student chooses music and drama given that she chooses a concert than it is that a student chooses a concert given that she chooses music and drama.
7. 0.4 or 40%
9. $P(\text{sum} = 7 \mid \text{numbers are different}) = 6/30 = 1/5$; $P(\text{numbers are different} \mid \text{sum} = 7) = 6/6 = 1$

Practice 7.2.2: Using Two-Way Frequency Tables, pp. 81–83

1.

Sport	Gender	
	Male	Female
Football	𝍸 II	II
Basketball	II	III
Baseball	I	I
Softball	I	III

3. $P(TEN \mid V) < P(V \mid TEN)$; this means the probability that a work-study student is in the tenth grade given that she has a volunteer position is less than the probability that a work-study student has a volunteer position given that she is in the tenth grade.
5. $P(TWELVE \mid P) < P(P \mid TWELVE)$; this means the probability that a work-study student is in the twelfth grade given that she has a paid job is less than the probability that a work-study student has a paid job given that she is in the twelfth grade.
7. $P(M \mid P) > P(P \mid M)$; M and P are dependent; $P(M) = 80/160 = 0.5$; $P(M \mid P) = 12/41 \approx 0.293$; $P(P) = 41/160 \approx 0.256$; $P(P \mid M) = 12/80 = 0.15$
9. $P(M \mid MU) > P(MU \mid M)$; M and MU are independent; $P(M) = 80/160 = 0.5$; $P(M \mid MU) = 20/41 \approx 0.488$; $P(MU) = 41/160 \approx 0.256$; $P(MU \mid M) = 20/80 = 0.25$