LESSON 9.6

Special Right Triangles

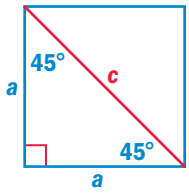
BEFORE You found side lengths of right triangles.

Now You'll use special right triangles to solve problems.

WHY? So you can find the distance a softball is thrown, as in Ex. 15.

Review Vocabulary
hypotenuse, p. 465
leg, p. 465

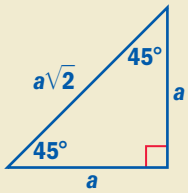
A diagonal of a square divides it into two 45°-45°-90° triangles. In such a triangle, the lengths of the legs are equal. Let a represent the length of each leg, and let c represent the length of the hypotenuse. By the Pythagorean theorem, $c^2 = a^2 + a^2 = 2a^2$, so $c = \sqrt{2a^2} = a\sqrt{2}$.

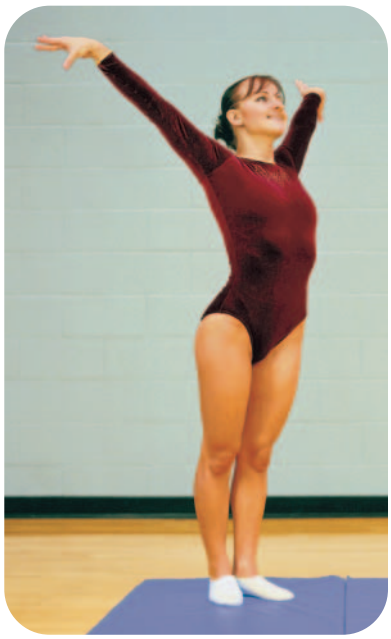


45°-45°-90° Triangle

Words In a 45°-45°-90° triangle, the length of the hypotenuse is the product of the length of a leg and $\sqrt{2}$.

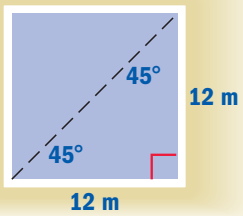
Algebra hypotenuse = leg • $\sqrt{2}$
$= a\sqrt{2}$





Example 1 *Using a 45°-45°-90° Triangle*

Gymnastics The mat used for floor exercises at a gymnastics competition is a square with a side length of 12 meters. A gymnast starts at one corner of the mat and does a tumbling routine along the diagonal to the opposite corner. To the nearest meter, how long is the gymnast's path?



Solution

The diagonal divides the mat into two 45°-45°-90° triangles. The diagonal is the hypotenuse of each of the triangles.

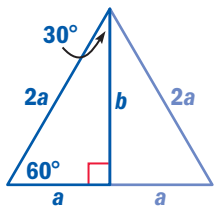
hypotenuse = leg • $\sqrt{2}$ — **Rule for 45°-45°-90° triangle**
$= 12 \cdot \sqrt{2}$ — **Substitute.**
≈ 17 — **Use a calculator.**

Answer The gymnast's path is about 17 meters long.

Note *Worthy*

You can organize information about right triangles in your notebook using a concept map like the one on p. 452.

30°-60°-90° Triangle You can divide an equilateral triangle in half as shown to make two 30°-60°-90° triangles. In the diagram, the equilateral triangle has side lengths of $2a$. Each right triangle has a hypotenuse of length $2a$ and a shorter leg of length a. Let b be the length of the longer leg. By the Pythagorean theorem, $(2a)^2 = a^2 + b^2$. Then $b^2 = 4a^2 - a^2 = 3a^2$, so $b = \sqrt{3a^2} = a\sqrt{3}$.



Study *Strategy*

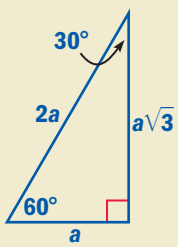
In a 30°-60°-90° triangle, the shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

30°-60°-90° Triangle

Words In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is the product of the length of the shorter leg and $\sqrt{3}$.

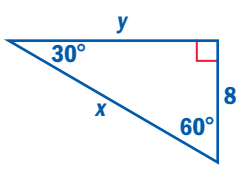
Algebra hypotenuse $= 2 \cdot$ shorter leg $= 2a$

longer leg $=$ shorter leg $\cdot \sqrt{3} = a\sqrt{3}$



Example 2 *Using a 30°-60°-90° Triangle*

Find the length x of the hypotenuse and the length y of the longer leg of the triangle.



The triangle is a 30°-60°-90° triangle. The length of the shorter leg is 8 units.

a. hypotenuse $= 2 \cdot$ shorter leg

$x = 2 \cdot 8$

$= 16$

Answer The length x of the hypotenuse is 16 units.

b. longer leg $=$ shorter leg $\cdot \sqrt{3}$

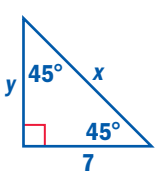
$y = 8\sqrt{3}$

Answer The length y of the longer leg is $8\sqrt{3}$ units.

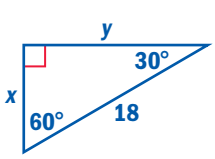
Checkpoint

Find the unknown lengths. Write your answer in simplest form.

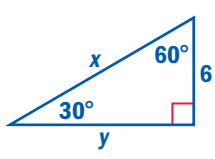
1.



2.

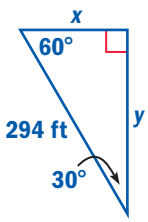


3.



Example 3 *Using a Special Right Triangle*

Architecture The base of the Massachusetts Institute of Technology's Building 66, an engineering laboratory, is approximately a 30°-60°-90° triangle. The length of the hypotenuse of the triangle is about 294 feet. Find, to the nearest foot, the lengths of the legs of the triangle.



Solution

You need to find the length of the shorter leg first.

1 Find the length x of the shorter leg.

$\text{hypotenuse} = 2 \cdot \text{shorter leg}$ — **Rule for 30°-60°-90° triangle**

$294 = 2x$ — **Substitute.**

$147 = x$ — **Divide each side by 2.**

2 Find the length y of the longer leg.

$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$ — **Rule for 30°-60°-90° triangle**

$y = 147\sqrt{3}$ — **Substitute.**

≈ 255 — **Use a calculator.**

Answer The length of the shorter leg of the triangle is 147 feet. The length of the longer leg is about 255 feet.

9.6 Exercises

More Practice, p. 811

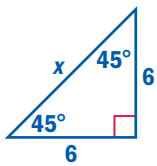
INTERNET eWorkbook Plus CLASSZONE.COM

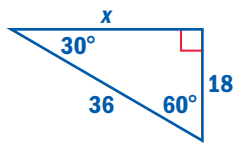
Guided Practice

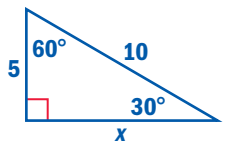
Vocabulary Check

1. Each leg of a 45°-45°-90° triangle has a length of 15 units. What is the length of the hypotenuse?
2. How is the length of the longer leg of a 30°-60°-90° triangle related to the length of the shorter leg?

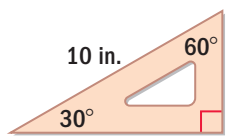
Skill Check **Find the unknown length. Write your answer in simplest form.**

3. 

4. 

5. 

6. **Graphic Arts** A graphic artist's tools include a 30°-60°-90° triangle. The hypotenuse of the triangle has a length of 10 inches. To the nearest inch, how long are the legs of the triangle?



Practice and Problem Solving

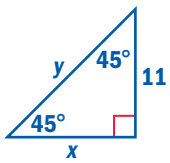
Homework Help

Example	Exercises
1	7–9, 14, 16–18
2	10–12, 16–18
3	13, 15

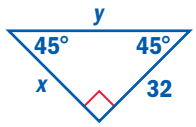
Online Resources CLASSZONE.COM
- More Examples
- eTutorial Plus

Find the unknown lengths. Write your answers in simplest form.

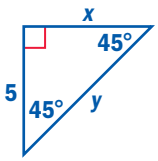
7.



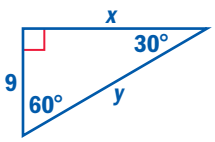
8.



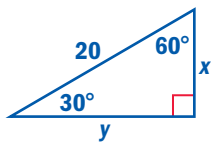
9.



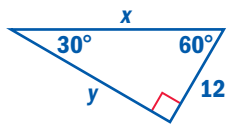
10.



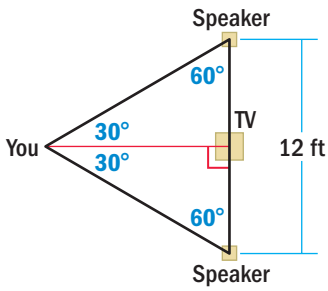
11.



12.



13. Speakers You connect a stereo system to your television set. The directions say that the speakers should be in line with your television and 12 feet apart as shown.

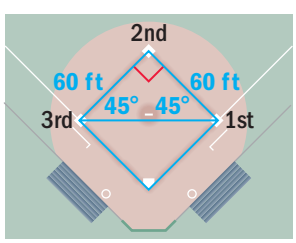


a. Find the distance between you and the television set to the nearest foot.

b. Find the distance between you and each speaker to the nearest foot.

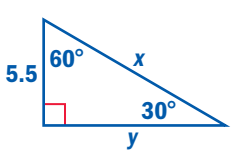
14. *Writing* Explain why any two 45°-45°-90° triangles are similar.

15. Softball The bases on a softball field form a square with a side length of 60 feet. You throw a softball from first base to third base. How far do you throw the softball? Round your answer to the nearest foot.

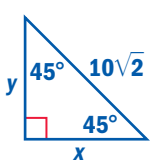


Find the unknown lengths. Write your answers in simplest form.

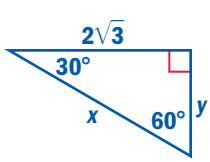
16.



17.



18.



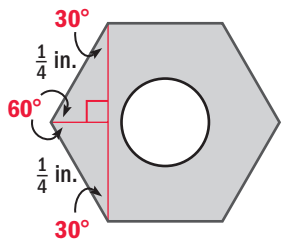
19. Extended Problem Solving There is a park in your town that is a square with a side length of 800 feet. You plan to walk from one corner of the square to the opposite corner.

a. Compare To the nearest foot, how much shorter is the distance from one corner to the opposite corner along the diagonal than the distance along two sides of the square?

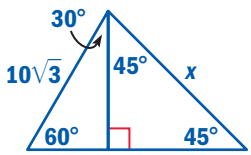
b. You walk at a rate of 3 miles per hour. Find your rate in feet per second.

c. Interpret To the nearest second, how much time would you save by walking along the diagonal rather than walking along two sides of the square?

20. Wrenches You must choose the right size wrench to tighten a nut. Each edge of the nut has a length of $\frac{1}{4}$ inch. You should choose a wrench size that is close to the distance across the nut from one edge to the opposite edge. Which wrench size should you use, $\frac{3}{8}$ inch, $\frac{7}{16}$ inch, or $\frac{1}{2}$ inch?



21. Challenge Find the value of x. Give your answer as a radical in simplest form.



Mixed Review

Solve the proportion. *(Lesson 6.2)*

22. $\frac{w}{7} = \frac{36}{42}$ **23.** $\frac{x}{10} = \frac{35}{50}$ **24.** $\frac{3}{4} = \frac{y}{52}$ **25.** $\frac{7}{12} = \frac{z}{105}$

26. Submarines A sailor on a submarine uses a periscope to view the surface of the ocean. The periscope's height h (in feet) above the surface and the distance d (in miles) that the sailor can see are related by the formula $h = \frac{d^2}{1.4}$. Suppose the periscope is at a height of 3 feet. To the nearest mile, how far can the sailor see? *(Lesson 9.1)*

Find the midpoint of the segment with the given endpoints. *(Lesson 9.5)*

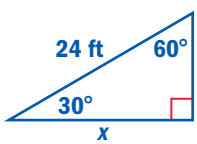
27. $(-3, 4), (-1, 6)$ **28.** $(8, -3), (-2, 7)$ **29.** $(4, -1.1), (-2.4, -1.7)$

Standardized Test Practice

30. Multiple Choice What is the value of x?

A. $\frac{12}{\sqrt{3}}$ ft **B.** 12 ft

C. $12\sqrt{3}$ ft **D.** $24\sqrt{3}$ ft



31. Multiple Choice Each leg of a 45°-45°-90° triangle has a length of 15 units. What is the length of the hypotenuse?

F. 7.5 units **G.** $\frac{15}{\sqrt{2}}$ units **H.** 15 units **I.** $15\sqrt{2}$ units

32. Short Response Explain how to find the area of the equilateral triangle shown.