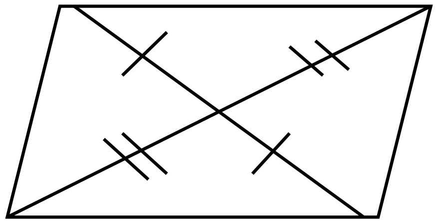
7.9—Proving a Quadrilateral is a Rhombus, Rectangle, or a Square

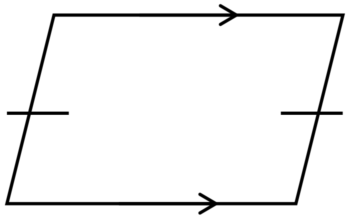
7.9 Day 1 Warm Up

For each figure, tell if there is enough information to prove that the quadrilateral is a parallelogram. If so, give the theorem or definition.

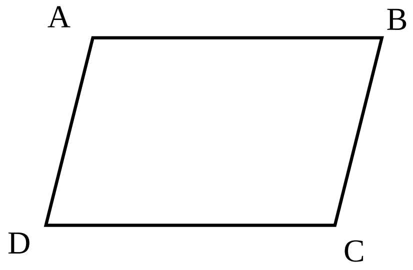
1.



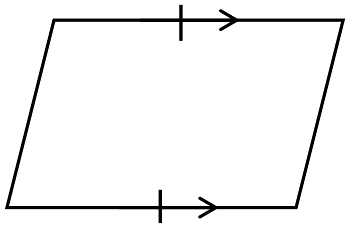
2.



3. $m\angle A + m\angle D = 180°$



4.

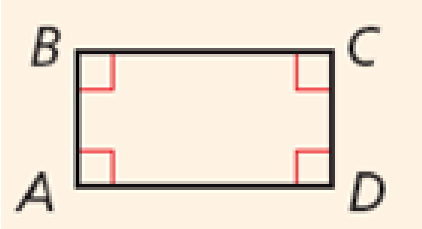


7.9—Proving a Quadrilateral is a Rhombus, Rectangle, or a Square

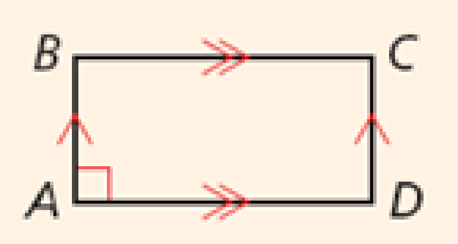
Objective: *Use properties of sides, angles, and diagonals of rhombus, rectangles, and squares.*

Ways to Prove that a Quadrilateral is a Rectangle:

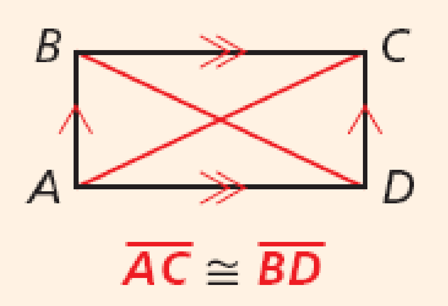
✓ Show that it has four right angles
(Definition of Rect.)



✓ Show that it is a parallelogram with one right angle.
(▱ w/ one rt. $\angle \longrightarrow$ Rect.)

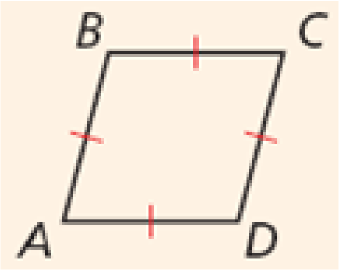


✓ Show that it is a parallelogram with diagonals that are congruent.
(▱ w/ diags $\cong \longrightarrow$ Rect.)

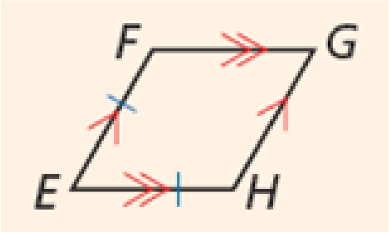


Ways to Prove that a Quadrilateral is a Rhombus:

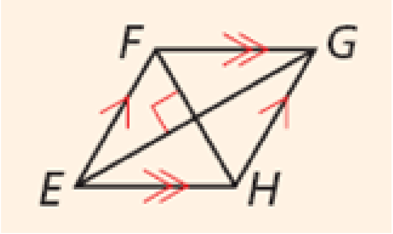
- ✓ Show that it has four congruent sides
  (Definition of Rhombus)



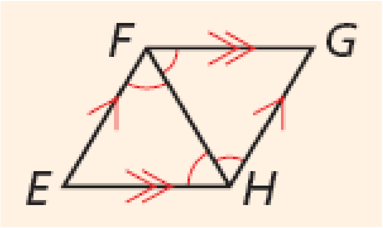
- ✓ Show that it is a parallelogram with one pair of consecutive sides congruent.
  (▱ w/ one pair cons. sides ≅ $\longrightarrow$ Rhombus)



- ✓ Show that it is a parallelogram with diagonals are perpendicular.
  (▱ w/ diags ⊥ $\longrightarrow$ Rhombus)



- ✓ Show that it is a parallelogram with a diagonal bisects the angles.
  (▱ w/ diag bisect ∠s $\longrightarrow$ Rhombus)

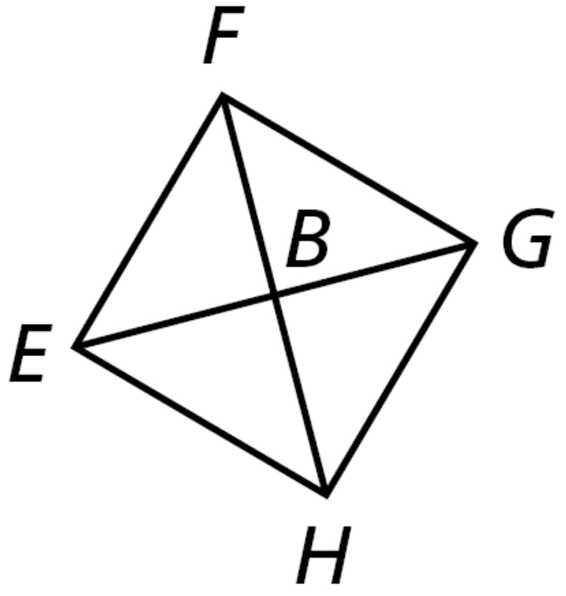


Examples:

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

1. **Given:** $\overline{EF} \cong \overline{FG}$, $\overline{EG} \perp \overline{FH}$

   Conclusion: EFGH is a rhombus.



The theorems for a rhombus are:

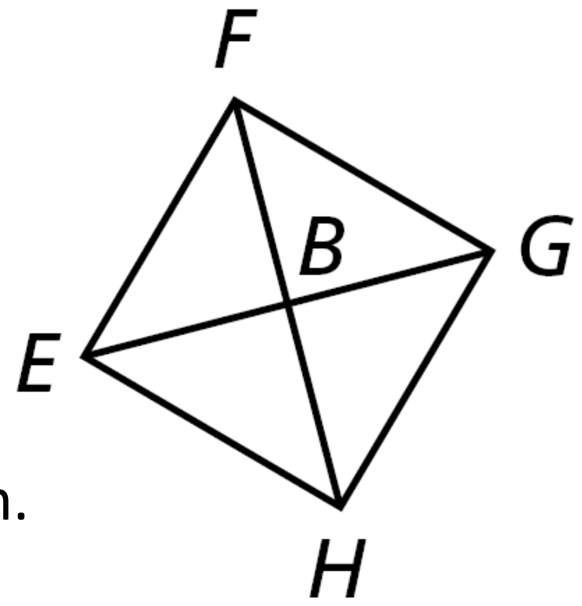
▱ w/ one pair cons. sides $\cong$ $\longrightarrow$ Rhombus or

▱ w/ diags $\perp$ $\longrightarrow$ Rhombus

However, to apply either theorem, you must first know that *EFGH* is a parallelogram, which can’t be proven. Therefore, the conclusion is not valid.

2. Given: $\overline{EB} \cong \overline{BG}, \overline{FB} \cong \overline{BH}, \overline{EG} \cong \overline{FH}, \triangle EBF \cong \triangle EBH$

Conclusion: EFGH is a square.



The diagonals bisect each other, so EFGH is a parallelogram.

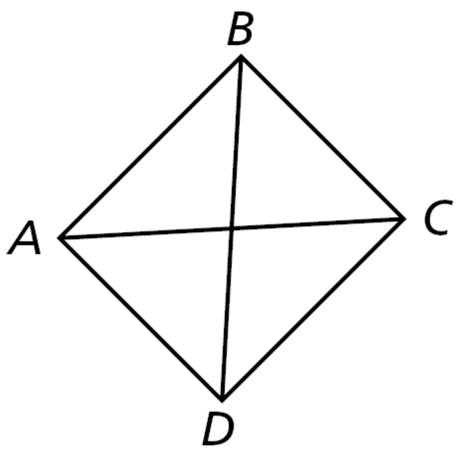
The diagonals are congruent, so EFGH is a rectangle.

Since $\Delta\mathrm{EBF} \cong \Delta\mathrm{EBH}$, $\overline{\mathrm{EF}} \cong \overline{\mathrm{EH}}$.
A pair of consecutive angles are congruent, so EFGH is a rhombus.

Since EFGH is a rectangle and a rhombus, it is a square.

3. Given: $\angle ABC$ is a right angle

Conclusion: *ABCD* is a rectangle.



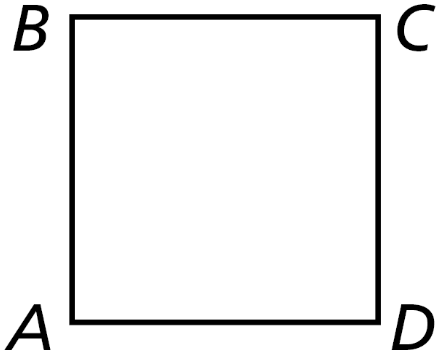
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

To apply this theorem, you need to know that ABCD is a parallelogram .

Therefore, the conclusion is not valid.

4. Given: $AB = BC = CD = DA$, $AC = BD$

Conclusion: *ABCD* is a square



All four sides are congruent, so ABCD is a rhombus by definition and also a parallelogram.

The diagonals are congruent, so ABCD is a rectangle.

Since ABCD is a rhombus and a rectangle, it is also a square.

7.9 Day 2—Special Quadrilaterals Coordinate Proofs

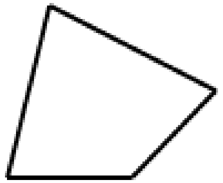
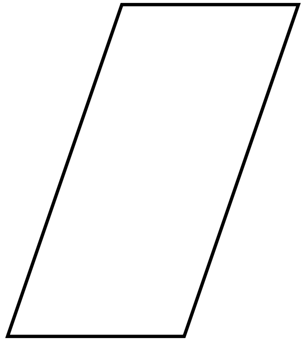
7.9 Day 2 Warm Up

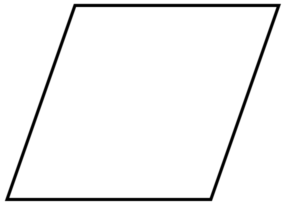
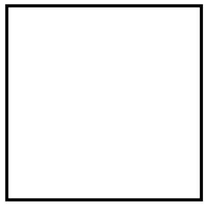
Use the points A(-3, 7) & B(5, -3) to find the following:

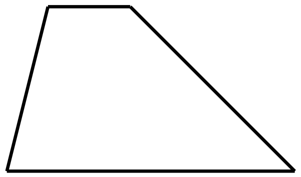
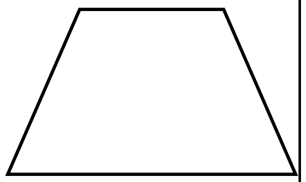
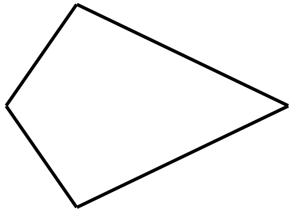
1. Slope **2. Midpoint** **3. Distance**

7.9 Day 2—Special Quadrilaterals Coordinate Proofs

Objective: Use properties of sides, angles, and diagonals to prove special quadrilaterals.

Shape	Sketch	Properties	Coordinate Proofs	Area
Quad.		• 4 sided polygon • Interior $\angle s$ add to = 360°		
Parallelogram		• Both pairs opp. sides ∥ • Both pairs opp. sides ≅ • Diags. bisect e. o. • Both pairs opp. $\angle s$ ≅ • Consec. $\angle's$ supp.	➢ Opp. sides same slope ➢ Opp. sides same distance ➢ Diags. same midpoint	$\boldsymbol{A = b \cdot h}$

Rhombus		• 4 sides ≅ • ▱ w/ diags. ⊥ • ▱ w/ diags are ∠ bisectors	➢ All 4 sides same distance ➢ ▱ (diags. same midpt.) **AND** diags slopes are opp. reciprocals	$\boldsymbol{A = \frac{1}{2} d_1 \cdot d_2}$
Rectangle		• 4 rt. $\angle s$ • ▱ w/ diags ≅	➢ Consec. sides slopes are opp. reciprocals ➢ ▱ (diags. same midpt.) **AND** diags. same distance	$\boldsymbol{A = b \cdot h}$
Square		• 4 sides ≅ • 4 rt. $\angle s$ • ▱ w/ diags. ⊥ • ▱ w/ diags ≅ • ▱ w/ diags are ∠ bisectors	➢ All sides same distance **AND** Consec. sides slopes are opp. reciprocals ➢ ▱ (diags. same midpt.) **AND** diags slopes are opp. reciprocals ➢ ▱ (diags. same midpt.) **AND** diags. same distance	$\boldsymbol{A = s^2}$

Trapezoid		• Exactly one pair of ∥ sides	➢ Only one pair opp. sides have same slope	$A = \frac{1}{2}(b_1 + b_2) \cdot h$
Isosceles Trapezoid		• One pair opp. sides ∥ • Legs are ≅ • Diags. are ≅	➢ Only one pair opp. sides have same slope **AND** ➢ Legs same distance **or** ➢ Diags same distance	
Kite		• 2 pairs consec. sides ≅ (opp. sides not ≅) • Diags are ⊥ • Only one pair opp. $\angle s$ ≅	➢ 2 pairs consec. sides have the same distance	$A = \frac{1}{2} d_1 \cdot d_2$

Use Coordinate Geometry to determine what kind of Parallelogram the coordinates of four vertices make.

____

diagonals bisect each other
(same midpoint)

____

diagonals are perpendicular
(slopes are opposite reciprocals)

____

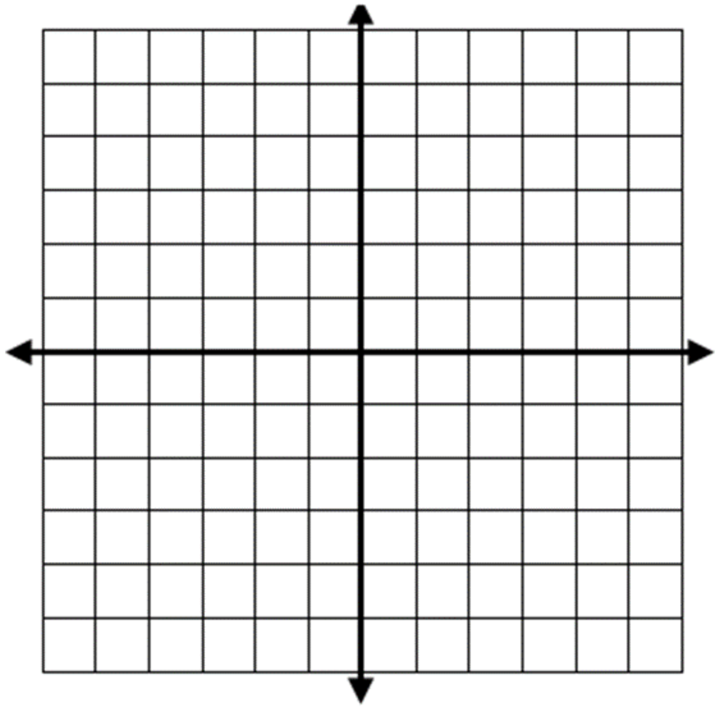
diagonals are congruent
(same distance)

(both)

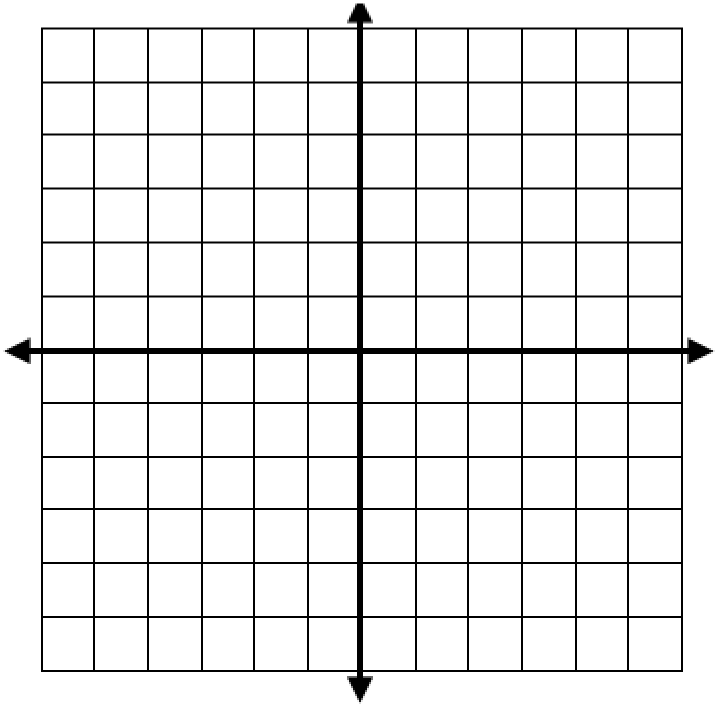
____

Example: Determine what kind of quadrilateral the four points make.

5. $M(-2, -1)$ $A(1, 3)$ $T(5, 0)$ $H(2, -4)$



6. $P(-1, 3)$ $Q(-2, 5)$ $R(0, 4)$ $S(1, 2)$



7. L(−1, 1)   M(1, 3)   N(3, 1)   O(1, −3)