## Rigid transformations

Student Activity Sheet 3; use with Exploring "Reflections and rotations"

1. In the diagram, $\Delta A^{\prime} B^{\prime} C^{\prime} C$ is a composition of two reflections across lines $m_{1}$ and $m_{2}$. Compare how $\triangle \mathrm{ABC}$ and $\Delta \mathrm{A} " \mathrm{~B} " \mathrm{C}$ " are oriented with respect to the intersecting lines. How can you tell from the images that $\Delta A^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{C}^{\prime}$ is not a translation of $\triangle \mathrm{ABC}$ ?
[EX2, page 2]


In a translation, each point in a figure moves the same distance in the same direction. So, the line segments connecting corresponding points would have to be the same length and parallel. In this case, the line segments connecting the corresponding vertices of $A B C$ and $\triangle A^{\prime} B^{\prime} '^{\prime} C^{\prime \prime}$ are not parallel or congruent. So, this transformation cannot be a translation.
2. Fill in the blanks to complete the following statements. [EX2, page 4]

A rotation of a point about a fixed point is a composite of two reflections of the point across intersecting lines. The point of intersection of the lines is the center of rotation.
3. Compare the orientation of $\triangle A B C$ and $\Delta A$ " $B$ " $C$ ". Do you think your observation is true for all rotations? [EX2, page 4]

On both triangles, the vertices go in a counterclockwise direction, from $A$ to $B$ to $C$. Therefore, they have the same orientation. In general, rotations preserve orientation.

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4. Use the diagram to write another definition for rotation. [EX2, page 6]


A rotation about a point $P$ maps $A$ to $A^{\prime \prime}$ so that $\angle A P A^{\prime \prime}$ is the angle of rotation and $P A=P A^{\prime \prime}$.

Use the terms provided to fill in the blanks in the statements below. [EX2, page 7]

| bisector | $4 x$ | rotation | transformation |
| :---: | :---: | :---: | :---: |
| perpendicular <br> bisector | reflection | translation | $x$ |

5. A rigid transformation preserves distance, angle measure, and area.
6. Point $P^{\prime}$ is a reflection of $\mathbf{P}$ across line $m$ if and only if $m$ is the perpendicular bisector of PP'.
7. A translation is a composition of two reflections over two parallel lines. If the distance between the lines is $2 x$, then the distance between the image and the pre-image is $4 x$.
8. A rotation is a composition of two reflections over two intersecting lines. If the measure of the angle of rotation is $2 x$, then the measure of the acute angle of the intersecting lines is $\boldsymbol{x}$.

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9. REINFORCE In the diagram, assume the measure of the acute angle between $m_{1}$ and $m_{2}$ is $80^{\circ}$, and $\mathrm{m} \angle \mathrm{APA}=(5 x-30)^{\circ}$. Solve for $\boldsymbol{x}$.
$m \angle A P A^{\prime \prime}$ is twice the measure of the acute angle formed by lines $\boldsymbol{m}_{1}$ and $\boldsymbol{m}_{2}$.
$5 x-30=2(80)$
$5 x-30=160$
$5 x=190$
$x=38$
10. REINFORCE $\operatorname{In}$ the diagram, assume PA $=(6 x+4)$ inches and PA" $=(7 x+1)$ inches. Solve for $x$.

Since $A^{\prime \prime}$ is a rotation of $A$ about $P, P A=P A "$.
$6 x+4=7 x+1$
$3=x$

