WORKSHEET 2.1 & 2.2

1.10.1 Remainder Theorem and Factor Theorem

Remainder Theorem:
When a polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

   (a) $x-1$ (b) $x-2$ (c) $x-3$

   (d) $x+1$ (e) $x+2$ (f) $x+3$

Factor Theorem:
If $x=a$ is substituted into a polynomial for x, and the remainder is 0, then $x-a$ is a factor of the polynomial.

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$, complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen ($x-a$) and remember to fully factor your result in each case.

1.10.1 Remainder Theorem and Factor Theorem (Answers)

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

(a) $x-1$

$\therefore a=1$

$f(1)=2(1)^3+3(1)^2-17(1)-30$

$f(1)=2+3-17-30$

$f(1)=-42$

(b) $x-2$

$a=2$

$f(a)=-36$

(c) $x-3$

$a=3$

$f(a)=0$

(d) $x+1$

$a=-1$

$f(a)=-12$

(e) $x+2$

$a=-2$

$f(a)=0$

(f) $x+3$

$a=-3$

$f(a)=-6$

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

From results → (c) $x-3$ and (e) $x+2$ are factors

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$ complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen $x-a$) and remember to fully factor your result in each case.

(c) $x-3$

$$
\begin{array}{r|llll}
 & 2x^2 & +9x & +10 & \\
\hline
x-3 & 2x^3 & +3x^2 & -17x & -30 \\
 & 2x^3 & -6x^2 & \downarrow & \downarrow \\
\cline{2-3}
 & & 9x^2 & -17x & \downarrow \\
 & & 9x^2 & -27x & \downarrow \\
\cline{3-4}
 & & & 10x & -30 \\
 & & & 10x & -30 \\
\cline{4-5}
 & & & & 0
\end{array}
$$

Result: $(x-3)(2x^2+9x+10)$

$(x-3)(2x+5)(x+2)$

(e) $x+2$

$$
\begin{array}{r|llll}
 & 2x^2 & -x & -15 & \\
\hline
x+2 & 2x^3 & +3x^2 & -17x & -30 \\
 & 2x^3 & +4x^2 & \downarrow & \downarrow \\
\cline{2-3}
 & & -x^2 & -17x & \downarrow \\
 & & -x^2 & -2x & \downarrow \\
\cline{3-4}
 & & & -15x & -30 \\
 & & & -15x & -30 \\
\cline{4-5}
 & & & & 0
\end{array}
$$

Result: $(x+2)(2x^2-x-15)$

$(x+2)(2x+5)(x-3)$

(Note: The results are the same just rearranged.)

1.10.2 Dividing Polynomials Practice

Complete the polynomial divisions below:

1. Without using long division, find each remainder:

   (a) $(2x^2 + 6x + 8) \div (x + 1)$

   (b) $(x^2 + 4x + 12) \div (x - 4)$

   (c) $(x^3 + 6x^2 - 4x + 3) \div (x + 2)$

   (d) $(3x^3 + 7x^2 - 2x - 11) \div (x - 2)$

2. Find each remainder:

   (a) $(2x^2 + x - 6) \div (x + 2)$

   (b) $(x^3 + 6x^2 - 4x + 2) \div (x + 1)$

   (c) $(x^3 + x^2 - 12x - 13) \div (x - 2)$

   (d) $(x^4 - x^3 - 3x^2 + 4x + 2) \div (x + 2)$

3. When $x^3 + kx^2 - 4x + 2$ is divided by $x + 2$ the remainder is 26, find *k*.

4. When $2x^3 - 3x^2 + kx - 1$ is divided by $x - 1$ the remainder is 2, find *k*.