Piecewise Functions

Training for a Race

Saundra is a personal trainer at a local gym. Earlier this year, three of her clients asked her to help them train for an upcoming 5K race. Though Saundra had never trained someone for a race, she developed plans for each of her clients that she believed would help them perform their best.

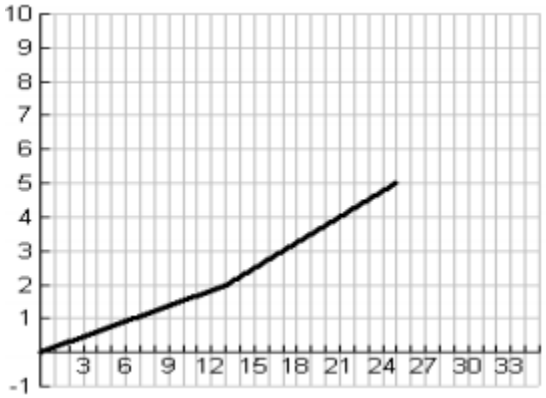
She wanted to see if her plans were effective, so when she attended the race to cheer them on, she collected data at regular intervals along the race. Her plan was to create graphs for each of the runners and compare their performances.

Since each had an individualized strategy, each runner ran a different plan during the race. One of her clients (Sue, the oldest one), was supposed to begin slowly, increasing over the first kilometer until she hit a speed which she believed she could maintain over the rest of the race.

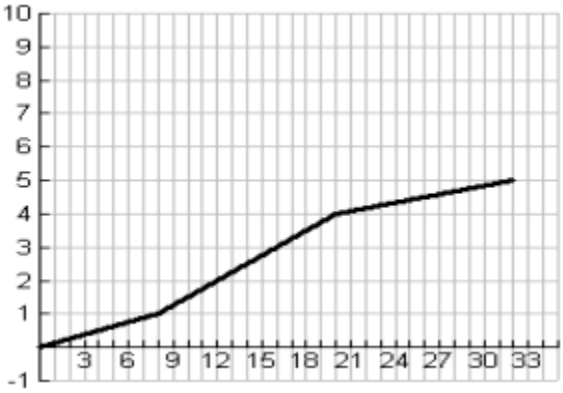
Her second client, Jim, was supposed to begin with a strong burst for the first kilometer, then slow to a steady pace until the final kilometer when he would finish with a strong burst.

Her third client, Jason, is a very experienced runner. His plan was to run at a steady pace for the first two kilometers, then run at his maximum speed for the final 3 kilometers.

Each of the clients came close to performing as they planned.



Graph 1



Graph 2

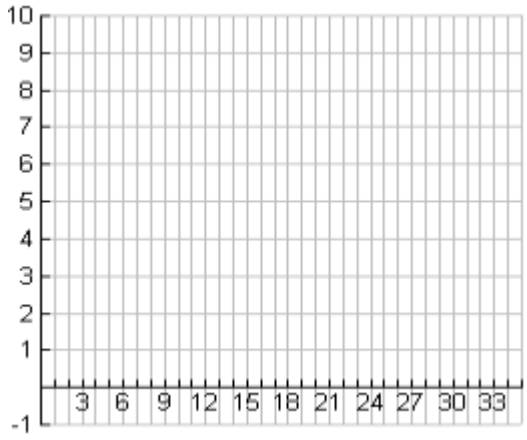
1. Saundra created graphs for two of the clients, but she set them aside without labeling the graphs. Now she cannot remember whose graphs she has. Can you identify the client based on these graphs? Explain how you know.

2. Describe how the runner in Graph #1 performed. For what distance did the runner increase speed, decrease speed, or maintain speed?

3. Compare the performance of the runner in Graph #2 to the runner in Graph #1.

4. Saundra found the data for her third client on her desk. Graph the data for this runner.

Time	Km
4:00	1
8:30	2
13:00	3
22:00	4
26:00	5



While you may be tempted to find a line that describes this data, a single line does not really show how the runner performed at each interval. A piecewise function is a graph that shows differences in specified intervals; that is, it is a graph with two or more pieces. The slope of the pieces may not be the same and even the shape of the pieces may not be the same.

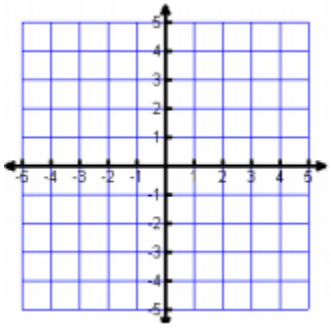
5. Connect the points in the third graph to show the "pieces" of different performance levels by the runner.

6. Using the third graph, write the equations of the "pieces," or segments, of the graph. Be sure to indicate the appropriate interval for each piece (for which x-values that equation is the correct graph).

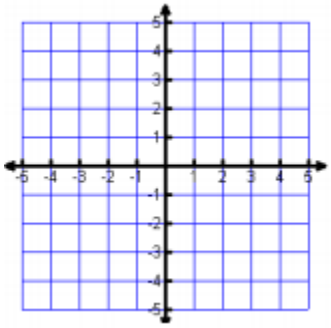
Manufacturing Moldings

Piecewise functions do not always have to be line segments. The "pieces" could be pieces of any kind of graph. Try to graph some of these piecewise functions. You may find it helpful to use what you already know about transformations of the parent functions

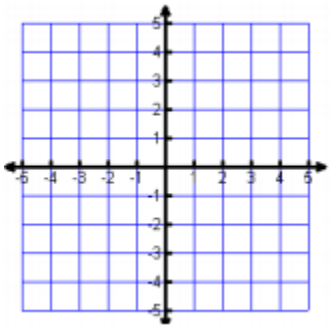
1. $f(x) = \begin{cases} x^2 + 4, & x < 0 \\ \sqrt{x} + 4, & x \geq 0 \end{cases}$



2. $f(x) = \begin{cases} |x| - 1, & x > -1 \\ x + 3, & x \leq -1 \end{cases}$



3. $f(x) = \begin{cases} 3, & -3 \leq x < -1 \\ x^2, & -1 \leq x < 1 \\ 3, & 1 \leq x \leq 3 \end{cases}$



4. In some manufacturing settings, machines can be programmed to make certain cuts based on piecewise functions the operator can define. What equations would you program into the machine to cut to create the following shape?