13.1 Right Triangle Trigonometry

Goals
- Use trigonometric relationships to evaluate trigonometric functions of acute angles.
- Use trigonometric functions to solve real-life problems.

Your Notes

VOCABULARY

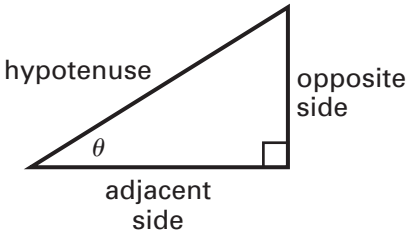
Solving a right triangle

Angle of elevation

Angle of depression

RIGHT TRIANGLE DEFINITION OF TRIGONOMETRIC FUNCTIONS

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.



$\sin\theta = \dfrac{\text{opp}}{\square}$ $\quad \cos\theta = \dfrac{\square}{\text{hyp}}$ $\quad \tan\theta = \dfrac{\text{opp}}{\square}$

$\csc\theta = \dfrac{\square}{\text{opp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\square}$ $\quad \cot\theta = \dfrac{\square}{\text{opp}}$

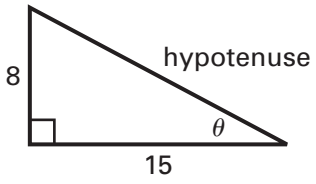
The abbreviations *opp*, *adj*, and *hyp* represent the lengths of the three sides of the right triangle. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

$\csc\theta = \dfrac{1}{\square}$ $\quad \sec\theta = \dfrac{1}{\square}$ $\quad \cot\theta = \dfrac{1}{\square}$

Your Notes

Example 1 ***Evaluating Trigonometric Functions***

Evaluate the six trigonometric functions of the angle θ shown in the right triangle.



Solution

The length of the hypotenuse is:

$\sqrt{8^2 + 15^2} = \sqrt{\quad} = \quad$ **Use Pythagorean theorem.**

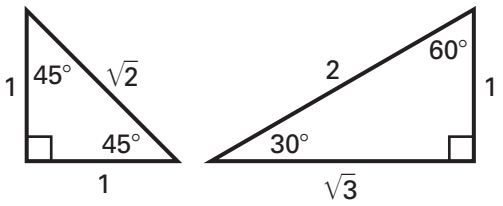
$\sin\theta = \dfrac{\text{opp}}{\quad} = \dfrac{\quad}{\quad}$ $\qquad$ $\cos\theta = \dfrac{\quad}{\text{hyp}} = \dfrac{\quad}{\quad}$

$\tan\theta = \dfrac{\text{opp}}{\quad} = \dfrac{\quad}{\quad}$ $\qquad$ $\csc\theta = \dfrac{\quad}{\text{opp}} = \dfrac{\quad}{\quad}$

$\sec\theta = \dfrac{\text{hyp}}{\quad} = \dfrac{\quad}{\quad}$ $\qquad$ $\cot\theta = \dfrac{\quad}{\text{opp}} = \dfrac{\quad}{\quad}$

✔ ***Checkpoint*** **Complete the following exercise.**

1. Evaluate the six trigonometric functions of the angles shown in the right triangles.

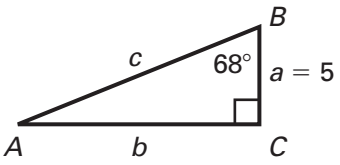


Your Notes

Example 2 ***Solving a Right Triangle***

Solve $\triangle ABC$.

Angles A and B are complementary angles, so $A = 90° - \_\_\_\_° = \_\_\_\_°$.



$\frac{b}{5} = \tan 68°$ $\qquad$ $\frac{c}{5} = \sec 68° = \frac{1}{\square} \approx \_\_\_\_$

$b \approx \_\_\_\_$ $\qquad$ $c \approx \_\_\_\_$

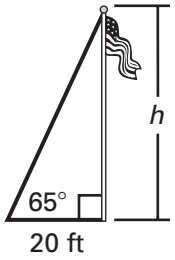
Example 3 ***Finding the Height of a Flag Pole***

You are measuring the height of a flag pole. You stand 20 feet from the base of the pole. You measure the angle of elevation from a point on the ground to the top of the pole to be 65°. Estimate the height of the pole to the nearest foot.

Solution

$\frac{\square}{\square} = \tan \_\_\_\_ \approx \_\_\_\_$

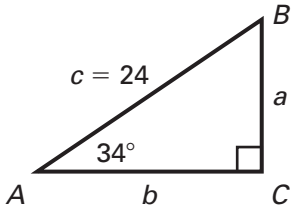
$h \approx \_\_\_\_$



Answer The height of the flag pole is about ____ feet.

✔ ***Checkpoint*** **Complete the following exercises.**

2. Solve $\triangle ABC$.



3. A cable to a radio tower makes an angle of 54° with the ground. If the cable is 240 feet, how far above the ground does it meet the tower? Round to the nearest foot.

Homework

13.1 Right Triangle Trigonometry

Goals
- Use trigonometric relationships to evaluate trigonometric functions of acute angles.
- Use trigonometric functions to solve real-life problems.

Your Notes

VOCABULARY

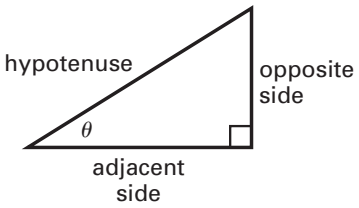
Solving a right triangle Finding all missing side lengths and angle measures of a right triangle

Angle of elevation The angle from a horizontal line through an object *B* to a line connecting *B* and a higher object *A*

Angle of depression The angle from a horizontal line through an object *A* to a line connecting *A* and a lower object *B*

RIGHT TRIANGLE DEFINITION OF TRIGONOMETRIC FUNCTIONS

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad \tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

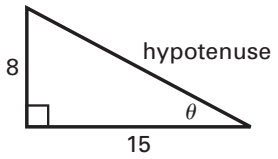
The abbreviations *opp*, *adj*, and *hyp* represent the lengths of the three sides of the right triangle. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta}$

Your Notes

Example 1 *Evaluating Trigonometric Functions*

Evaluate the six trigonometric functions of the angle θ shown in the right triangle.



Solution

The length of the hypotenuse is:

$\sqrt{8^2 + 15^2} = \sqrt{289} = 17$ **Use Pythagorean theorem.**

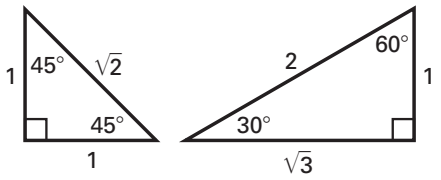
$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$ $\quad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$

✔ ***Checkpoint*** **Complete the following exercise.**

1. Evaluate the six trigonometric functions of the angles shown in the right triangles.



$\csc 30° = \sec 60° = 2$ $\quad \csc 45° = \sec 45° = \sqrt{2}$

$\cot 30° = \tan 60° = \sqrt{3}$ $\quad \sin 30° = \cos 60° = \frac{1}{2}$

$\sec 30° = \csc 60° = \frac{2\sqrt{3}}{3}$ $\quad \sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$

$\sin 45° = \cos 45° = \frac{\sqrt{2}}{2}$ $\quad \tan 30° = \cot 60° = \frac{\sqrt{3}}{3}$

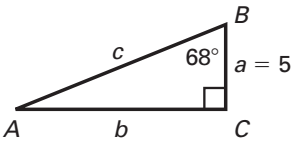
$\tan 45° = \cot 45° = 1$

Your Notes

Example 2 *Solving a Right Triangle*

Solve $\triangle ABC$.

Angles A and B are complementary angles, so $A = 90° - \underline{68}° = \underline{22}°$.



$\frac{b}{5} = \tan 68°$ $\qquad$ $\frac{c}{5} = \sec 68° = \frac{1}{\cos 68°} \approx \underline{2.669}$

$b \approx \underline{12.38}$ $\qquad$ $c \approx \underline{13.3}$

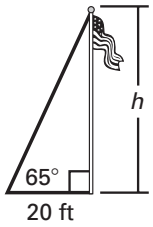
Example 3 *Finding the Height of a Flag Pole*

You are measuring the height of a flag pole. You stand 20 feet from the base of the pole. You measure the angle of elevation from a point on the ground to the top of the pole to be 65°. Estimate the height of the pole to the nearest foot.

Solution

$\frac{h}{20} = \tan \underline{65°} \approx \underline{2.145}$

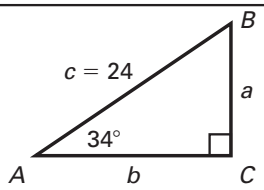
$h \approx \underline{43}$



Answer The height of the flag pole is about $\underline{43}$ feet.

✔ ***Checkpoint*** **Complete the following exercises.**

2. Solve $\triangle ABC$.



$B = 56°$, $a \approx 13.4$, $b \approx 19.9$

3. A cable to a radio tower makes an angle of 54° with the ground. If the cable is 240 feet, how far above the ground does it meet the tower? Round to the nearest foot.

about 194 feet

Homework