

# Logarithm Application Worksheet

1) **Healing of Wounds** The normal healing of wounds can be modeled by an exponential function. If  $A_0$  represents the original area of the wound and if  $A$  equals the area of the wound after  $n$  days, then the formula

$$A = A_0 e^{-0.35n}$$

describes the area of a wound on the  $n$ th day following an injury when no infection is present to retard the healing. Suppose a wound initially had an area of 100 square centimeters.

- If healing is taking place, how large should the area of the wound be after 3 days?
- How large should it be after 10 days?
- How many days will it take before the wound is 11 square centimeters?

2) **Response to TV Advertising** The percent of  $R$  viewers who respond to a television commercial for a new product after  $t$  days is found by using the formula

$$R = 70 - 100e^{-0.2t}$$

- What percent is expected to respond after 10 days?
- How many days until 40% of the viewers have responded?

3) **Optics:** If a single pane of glass obliterates 10% of the light though it. If  $P$  is the percent of light that passes though and  $n$  is the number of successive panes of panes. Find the number of panes of glass needed to successfully block 50% of the light given the equation below.

$$P = 100e^{-.1n}$$

- What percent of the light is blocked by 4 panes of glass?

4) If Tanisha has \$100 to invest at 8% per annum compounded monthly, how long will it be before she has \$150 if the money is compounded continuously?

b) What rate would Tanisha need to invest her money in order to make \$200 in 7 years and her money is compounded continuously?

9) A) **Radioactive Decay** The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years?

$n$  = number of half-lives       $t$  = number of years

$$y = A \left(\frac{1}{2}\right)^{\frac{t}{n}}$$

B) How many years until 2 grams are left?

**11) Population of an Endangered Species** Often environmentalists will capture an endangered species and transport the species to a controlled environment where the species can produce offspring and regenerate its population. Suppose 6 American Bald Eagles are captured and transported to Montana and set free. Based on experience, the environmentalists model

$$P(t) = \frac{500}{1 + 83.33e^{-0.162t}}$$

- (a) What is the predicted population of the American Bald Eagle in 20 years?
- (b) When will the population be 300?

**10) Radioactivity from Chernobyl** After the release of radioactive material into the atmosphere from a nuclear power plant at Chernobyl (Ukraine) in 1986, the hay in Austria was contaminated by iodine-131 (half-life 8 years.) If it is all right to feed the hay to cows when 10% of the iodine-131 remains, how long do the farmers need to wait to use this hay?

$$\% = \left(\frac{1}{2}\right)^{\left(\frac{1}{n}\right)t}$$

**Extra credit: Cooling Time of a Pizza**

A pizza baked at 450°F is removed from the oven at 5 pm into a room that is a constant 70°F. After 5 min the pizza is 300°F.

- a) Find k first.
- b) Then at what time can you eat the pizza if you want the pizza to be 135°F?

$$U(t) = T + (u_0 - T)e^{kt}$$

T=room temp  
 U<sub>0</sub>= initial temp pizza  
 t = time ; k = cooling constant

Solutions Manual: Use to check answers, not to copy solutions

1) a)  $34.9938 \text{ cm}^2$

b)  $3.01924 \text{ cm}^2$

c)  $\frac{11}{100} = \frac{100e^{-.35n}}{100}$

$.11 = e^{-.35n}$

$\ln .11 = \ln e^{-.35n}$

$\frac{\ln .11}{-.35} = \frac{-.35n}{-.35} (\ln e)$

$6.31 \text{ days} = n$

9) A)  $Y = 10\left(\frac{1}{2}\right)^{\frac{t}{1690}}$   
 $Y = 10\left(\frac{1}{2}\right)^{\frac{t}{1690}(50)}$   
 $= 10\left(\frac{1}{2}\right)^{\frac{5t}{1690}}$   
 $= 9.797 \text{ grams}$

B)  $\frac{2}{10} = \frac{10\left(\frac{1}{2}\right)^{\frac{t}{1690}}}{10}$   
 $.2 = \left(\frac{1}{2}\right)^{\frac{t}{1690}}$   
 $\log .2 = \frac{1}{1690} t (\log \frac{1}{2})$   
 $\log .2 = \frac{1}{1690} t (\log \frac{1}{2})$

$2.321 = \frac{1}{1690} t$

$3924 \text{ yrs} = t$

2) A)  $R = 70 - 100e^{-.2(10)}$   
 $= 70 - 100e^{-2}$   
 $= 70 - \frac{100}{e^2}$   
 $= 70 - 13.5335$   
 $= 56.4665 \%$

B)  $\frac{40}{70} = \frac{70 - 100e^{-.2t}}{70}$   
 $\frac{-30}{-100} = \frac{-100e^{-.2t}}{-100}$   
 $.3 = e^{-.2t}$   
 $\ln .3 = \ln e^{-.2t}$   
 $-1.2039 = -.2t$

$6.0199 = t$   
 days

3b) 67.0320%

11) c)  $P(t) = \frac{500}{1 + 83.33e^{-.162(20)}}$   
 $= \frac{500}{4.2635}$

$117.2739 \text{ eagles}$

d)  $\frac{300}{1} = \frac{500}{1 + 83.33e^{-.162t}}$

$\frac{300(1 + 83.33e^{-.162t})}{300} = \frac{500}{300}$

$1 + 83.33e^{-.162t} = 1.6$

$83.33e^{-.162t} = .6$

$\frac{83.33e^{-.162t}}{83.33} = \frac{.6}{83.33}$

$\ln e^{-.162t} = \ln .008$

$-.162t = -4.828$

$29.8 \text{ yrs} = t$

3) a)  $\frac{50}{100} = \frac{100e^{-.1n}}{100}$   
 $.5 = e^{-.1n}$   
 $\ln(.5) = \ln e^{-.1n}$   
 $-.6934 = -.1n$

$6.9314 \text{ panes}$

4)  $A = P\left(1 + \frac{r}{n}\right)^{nt}$   
 $150 = 100\left(1 + \frac{.08}{12}\right)^{12t}$   
 $\frac{150}{100} = \frac{100(1.006)^{12t}}{100}$   
 $1.5 = 1.006^{12t}$   
 $\log 1.5 = \frac{12t \log 1.006}{\log 1.006}$   
 $\frac{0.1761}{12} = \frac{12t}{12}$   
 $5.085 \text{ yrs} = t$

$A = Pe^{rt}$   
 $\frac{150}{100} = \frac{100e^{.08t}}{100}$   
 $1.5 = e^{.08t}$   
 $\ln 1.5 = \ln e^{.08t}$   
 $\frac{\ln 1.5}{.08} = \frac{.08t}{.08}$   
 $5.068 = t$   
 yrs

$A = Pe^{rt}$   
 $\frac{200}{100} = \frac{100e^{r(7)}}{100}$   
 $2 = e^{7r}$   
 $\ln 2 = 7r(\ln e)$   
 $\frac{\ln 2}{7} = \frac{7r}{7}$   
 $.0990 = r$   
 $9.90 \%$

10)  $\% = \left(\frac{1}{2}\right)^{\frac{t}{8}}$   
 $.10 = \frac{1}{2}^{\frac{t}{8}}$   
 $\log .10 = \log .5^{\frac{t}{8}}$   
 $\frac{\log .10}{\log .5} = \frac{1}{8} t (\log .5)$   
 $3.321 = \frac{1}{8} t$   
 $26.575 \text{ yrs} = t$

Extra Credit: (must have work to get credit)  
 17.58 min or 5:18pm