Transformational Geometry

Onto

- A function f:D->T is *onto,* if for every Y in T, there is a X in the domain, so that f(X)=Y

1-1

- A function f:D->T is *one-to-one,* if $X1 \neq X2$ implies that $f(X1) \neq f(x2)$.
- Another way to say this uses the contrapositive:
  if f(X1) = f(X2) then X1 = X2

Distance Preserving

- A function $f:R^2 \to R^2$ is a *distance preserving function if for any points A and B, the distance between A and B is the same as the distance between their images f(A) and f(B), ie.* |AB|=|f(A)f(B)|
- (check with the distance formula)

Isometry
(same measure)

- A function with all three properties:
  - 1-1
  - Onto
  - Distance Preserving.
- Other names are rigid motions or Euclidean motions.

Three Point Theorem

- In the Euclidean plan, the images of three noncollinear points completely determine an isometry. *In other words, if we know the outputs for three noncollinear points, A, B, C, we can figure out what the isometry does to any point X.*

Theorem

- An isometry preserves collinearity. Three points that are collinear will still be collinear after going through an isometry
- Use the Triangle Inequality!

Proof

- For any three points A, B, C, we have $|AB|+|BC|\geq|AC|$
- If the three points are collinear, there will be one way to list the points along their line that makes this an equality.
- Suppose that A, B, C is that correct ordering. Because the isometry f preserves distances, we have: $|f(A)f(B)|+|f(B)f(C)|=|f(A)f(C)|$

Therefore, the three points f(A), f(B) and f(C) must also be collinear.

Theorem

- An isometry preserves betweeness. In other words, if point B is between points A and C along a line, then point f(B) will be between f(A) and f(C) along their line.

Proof

- By previous theorem, we know that f(B) is collinear with f(A) and f(C). Suppose that f(A) is between the other two points. This makes $|f(A)f(B)|+|f(B)f(C)|>|f(A)f(C)|$
- However, because B is between A and C, we know that $|AB|+|BC|=|AC|$ which implies that $|f(A)f(B)|+|f(B)f(C)|=|f(A)f(C)|$
- Hence f(A) cannot be the middle point.
- A similar argument shows that f(C) cannot be the middle point either, leaving f(B) in the middle.

Isometries

- Under an isometry, the image of
- A line segment is congruent to a line segment
- A triangle is congruent to a triangle
- An angle is congruent to an angle
- A circle is congruent to a circle.
- *This is another way to study congruence.*

Composition of isometries

- g•f(x)=g(f(x)) $x \to f(x) \to g(f(x))$
- Of course it is necessary for the output of f to be a legitimate input for g, so that f(x) is in the domain of g. Otherwise the composition is undefined.
- For composition of isometries in the plane, any output is a point on the plane and can serve as the input of the next function.

Isometries

- In the Euclidean plane, there are only four types of isometry; translations, rotations, reflections, and glide reflections.