



 A function f:D->T is onto, if for every Y in T, there is a X in the domain, so that f(X)=Y

1-1

- A function f:D->T is *one-to-one*, if X1≠X2 implies that f(X1)≠f(x2).
- Another way to say this uses the contrapositive: if f(X1) = f(X2) then X1 = X2

Distance Preserving

- A function f:R²->R² is a distance preserving function if for any points A and B, the distance between A and B is the same as the distance between their images f(A) and f(B), ie. |AB|=|f(A)f(B)|
- (check with the distance formula)

Isometry

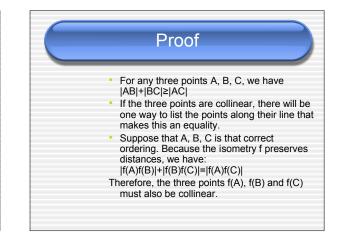
- A function with all three properties:
 - = 1-1
 - Onto
 - Distance Preserving.
- Other names are rigid motions or Euclidean motions.

Three Point Theorem

 In the Euclidean plan, the images of three noncollinear points completely determine an isometry. In other words, if we know the outputs for three noncollinear points, A, B, C, we can figure out what the isometry does to any point X.

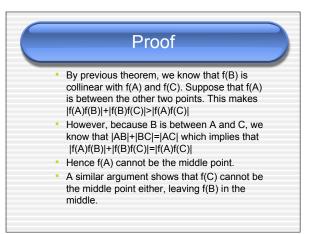
Theorem

- An isometry preserves collinearity. Three points that are collinear will still be collinear after going through an isometry
- Use the Triangle Inequality!



Theorem

 An isometry preserves betweeness. In other words, if point B is between points A and C along a line, then point f(B) will be between f(A) and f(C) along their line.



Isometries

- Under an isometry, the image of
- A line segment is congruent to a line segment
- A triangle is congruent to a triangle
- An angle is congruent to an angle
- A circle is congruent to a circle.
- This is another way to study congruence.

Composition of isometries

- $g \bullet f(x) = g(f(x))$ $x \to f(x) \to g(f(x))$
- Of course it is necessary for the output of f to be a legitimate input for g, so that f(x) is in the domain of g. Otherwise the composition is undefined.
- For composition of isometries in the plane, any output is a point on the plane and can serve as the input of the next function.

Isometries

• In the Euclidean plane, there are only four types of isometry; translations, rotations, reflections, and <u>glide reflections</u>.