Exponential and Logarithmic Functions & their Graphs

Exponential Functions and Graphs

Definition of an Exponential Function:

The **exponential function** with base is defined by For all real numbers , the function defined by

where , and is any real number. is called the **natural exponential function.**

Notice that the base of the exponential function is required to be positive and cannot be equal to 1.

Properties of an exponential function:

For all positive real numbers , the exponential function defined by has the following properties:

1. has the set of real numbers as its domain.
2. has the set of positive real numbers as its range.
3. has a graph with -intercept of .
4. has a graph asymptotic to the -axis.
5. is a one-to-one function.
6. is an increasing function if . See **Figure A.**
7. is a decreasing function if . See **Figure B.**

Figure A

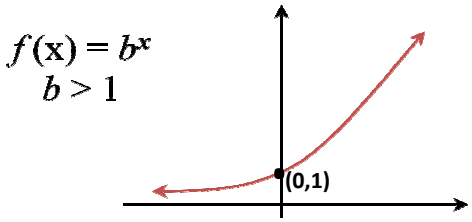
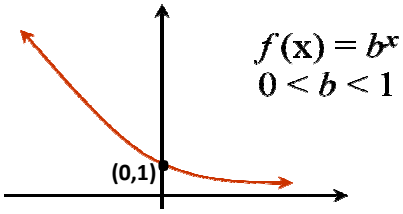


Figure B



Logarithmic Functions and Graphs

Definition of Logarithmic Function:

The **logarithmic function** with base is defined by

where is a positive constant, , and is any *positive* real number.

Note: Logarithmic functions are inverses of the corresponding exponential functions.

Properties of a logarithmic function:

For all positive real numbers , the function defined by has the following properties:

1. has the set of positive real numbers as its domain.
2. has the set of real numbers as its range.
3. has a graph with an -intercept of .
4. has a graph asymptotic to the -axis.
5. is a one-to-one function.
6. is an increasing function if . See **Figure C.**
7. is a decreasing function if . See **Figure D.**

Figure C

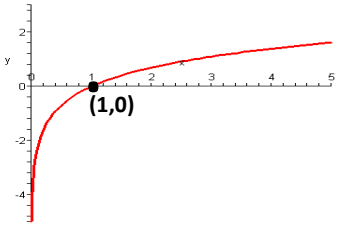
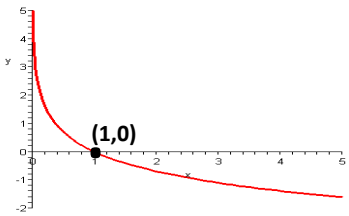


Figure D



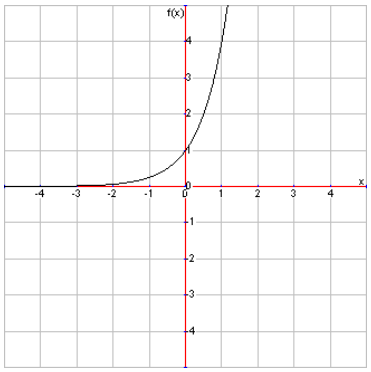
Practice Problems

Sketch graphs for each of the following exponential and logarithmic functions and label the intercepts:

1. $f(x) = 4^x$
2. $f(x) = 1 + e^{-x}$
3. $f(x) = \ln(x - 1)$
4. $f(x) = -\log x$
5. $f(x) = \log_3 x$
6. $f(x) = -\log_3 x$
7. $f(x) = -2 + \log_3 x$
8. $f(x) = 2 + \log_3 x$
9. $y = \log_2(x - 4)$
10. $y = \log_4(x + 1)$
11. $y = \ln(x - 3)$
12. $y = -\ln(x + 2)$
13. $y = 5 - \ln x$
14. $y = 3 + \ln x$

Practice Problems Answers

1.



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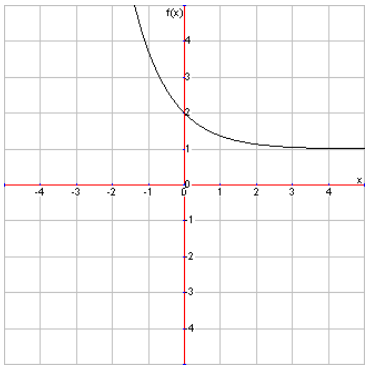

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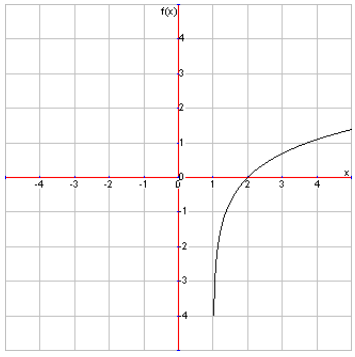

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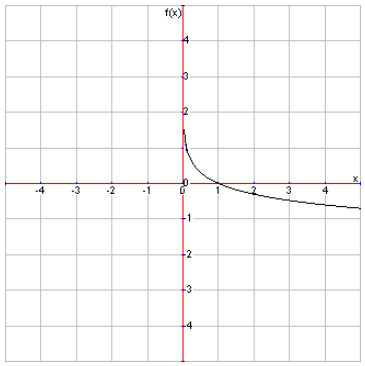

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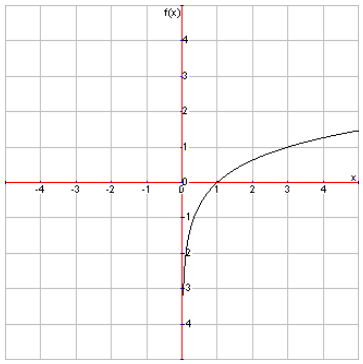

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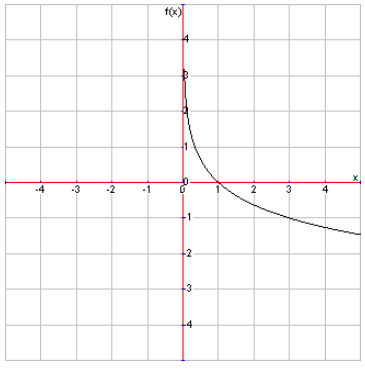

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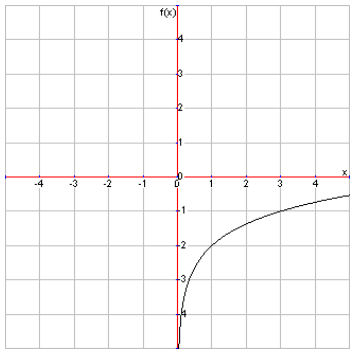

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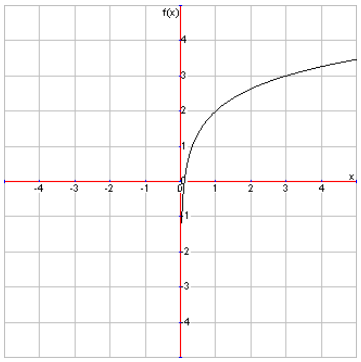

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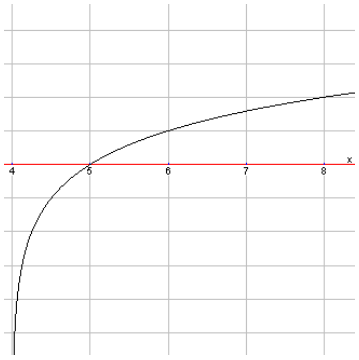

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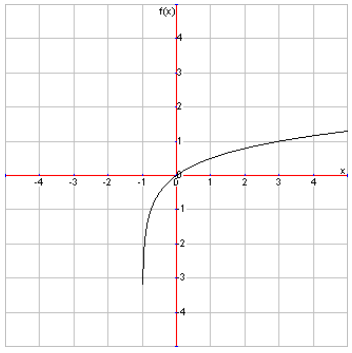

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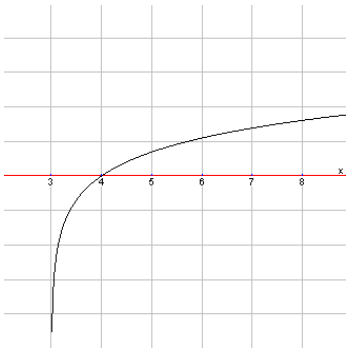

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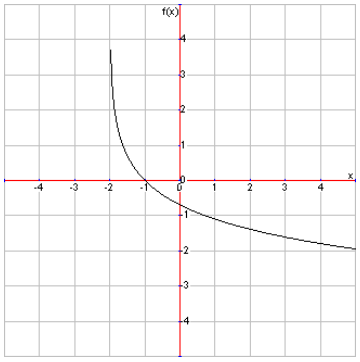

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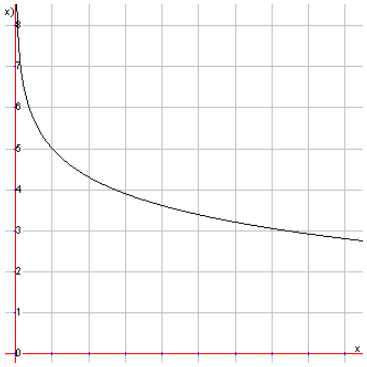

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