EXPONENTIAL GROWTH AND DECAY

GRAPHS OF EXPONENTIAL FUNCTIONS

> An exponential function is an equation of the form $y = ab^x$ (with $b \geq 0$).
>
> In many cases "*a*" represents a starting or initial value, "*b*" represents the multiplier or growth/decay factor, and "*x*" represents the time.

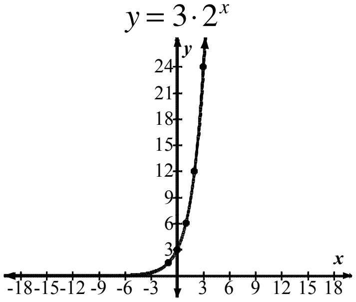
Example 1 Graph $y = 3 \cdot 2^x$

Make a table of values.

x	-1	0	1	2	3
y	1.5	3	6	12	24

Plot the points and connect them to form a smooth curve.

$y = 3 \cdot 2^x$



This is called an **increasing** exponential curve.

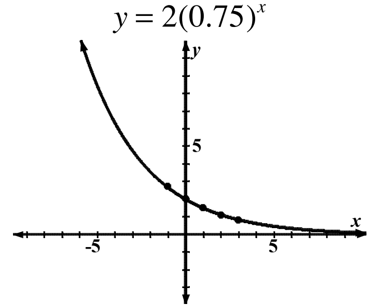
Example 2 Graph $y = 2(0.75)^x$

Make a table of values using a calculator.

x	-1	0	1	2	3
y	2.7	2	1.5	1.1	0.8

Plot the points and connect them to form a smooth curve.

$y = 2(0.75)^x$



This is called a **decreasing** exponential curve.

Problems

Make a table of values and draw a graph of each exponential function.

1. $y = 4(0.5)^x$
2. $y = 2(3)^x$
3. $y = 5(1.2)^x$
4. $y = 10(\frac{2}{3})^x$

SOLVING EXPONENTIAL GROWTH AND DECAY PROBLEMS

Example 1

Movie tickets now average \$9.75 a ticket, but are increasing 15% per year. How much will they cost 5 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 9.75$. The multiplier b is always found by adding the percent increase (as a decimal) to the number "one," so $b = 1 + 0.15 = 1.15$. The time is $x = 5$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 9.75(1.15)^5 \approx 19.61$. In five years movie tickets will average about \$19.61.

Example 2

A powerful computer is purchased for \$2000, but on the average loses 20% of its value each year. How much will it be worth 4 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 2000$. In this case the value is decreasing so multiplier b is always found by subtracting the percent decrease from the number "one," so $b = 1 - 0.2 = 0.8$. The time is $x = 4$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 2000(0.8)^4 = 819.2$. In four years the computer will only be worth \$819.20.

Example 3

Dinner at your grandfather's favorite restaurant now costs \$25.25 and has been increasing steadily at 4% per year. How much did it cost 35 **years ago** when he was courting your grandmother?

The equation is the same as above and $a = 25.25$, $b = 1.04$, but since we want to go back in time, $x = -35$. **A common mistake is to think that** $b = 0.96$. The equation is $y = ab^x = 25.25(1.04)^{-35} \approx 6.40$.

Example 4

If a gallon of milk costs \$3 now and the price is increasing 10% per year, how long before milk costs \$10 a gallon?

In this case we know the starting value $a = 3$, the multiplier $b = 1.1$, the final value $y = 10$, but not the time *x*. Substituting into the equation we get $3(1.1)^x = 10$. To solve this, you will probably need to guess and check with your calculator. Doing so yields $x \approx 12.6$ years. In Algebra 2 you will learn to solve these equations without guess and check.

Problems

5. The number of bacteria present in a colony is 180 at 12 noon and the bacteria grows at a rate of 22% per hour. How many will be present at 8 p.m.?

6. A house purchased for \$226,000 has lost 4% of its value each year for the past five years. What is it worth now?

7. A 1970 comic book has appreciated 10% per year and originally sold for \$0.35. What will it be worth in 2010?

8. A Honda Accord depreciates at 15% per year. Six year ago it was purchased for \$21,000. What is it worth now?

9. Inflation is at a rate of 7% per year. Today Janelle's favorite bread costs \$3.79. What would it have cost ten years ago?

10. Ryan's motorcycle is now worth \$2500. It has decreased in value 12% each year since it was purchased. If he bought it four years ago, what did it cost new?

11. The cost of a High Definition television now averages \$1200, but the cost is decreasing about 15% per year. In how many years will the cost be under \$500?

12. A two-bedroom house in Nashville is worth \$110,000. If it appreciates at 2.5% per year, when will it be worth \$200,000?

13. Last year the principal's car was worth \$28,000. Next year it will be worth \$25, 270. What is the annual rate of depreciation? What is the car worth now?

14. A concert has been sold out for weeks, and as the date of the concert draws closer, the price of the ticket increases. The cost of a pair of tickets was \$150 yesterday and is \$162 today. Assuming that the cost continues to increase at this rate:

    a. What is the daily rate of increase? What is the multiplier?

    b. What will be the cost one week from now, the day before the concert?

    c. What was the cost two weeks ago?

15. AN APPLICATION: CHOOSING A CAR

    Most cars decrease in value after you leave the dealer. However, some cars are now considered "classics" and actually increase in value. You have the choice of owning two cars: A 2006 Mazda Maita which is worth \$19,000 but is depreciating 10% per year, or a classic 1970 Ford Mustang which is worth \$11,500 and is increasing in value by 6% each year. Your tasks:

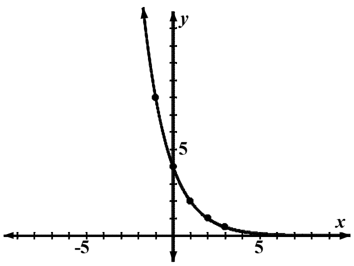
    a. Write an equation to represent the value of each car over time.

    b. Create tables and draw a graph to represent the value of each car for ten years on the same set of axes.

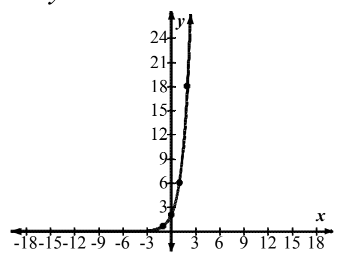
    c. Use your graph to determine approximately when the Mazda and the Ford have the same value.

Answers

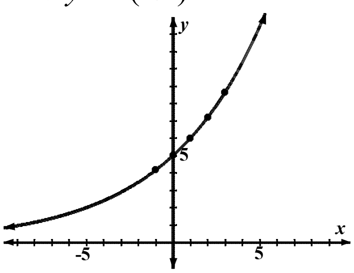
1. $y = 4\left(\frac{1}{2}\right)^x$



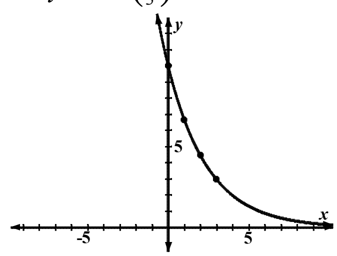
2. $y = 2 \cdot 3^x$



3. $y = 5(1.2)^x$



4. $y = 10\left(\frac{2}{3}\right)^x$



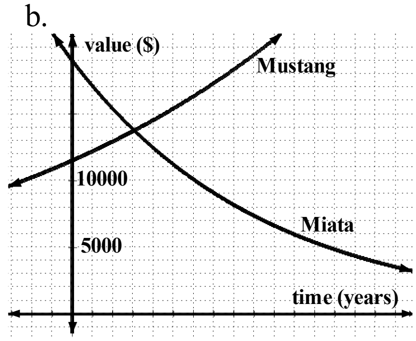
5. ≈ 883
6. $184,274
7. $15.84
8. $7920
9. $1.92
10. $4169
11. ≈ 5 years
12. ≈ 24 years
13. 5%, $26,600

14a. 8%, 1.08
b. $277.64
c. $55.15

15a. M: $y = 19000(.9)^x$

F: $y = 11500(1.06)^x$

b.



c. about 3 years