Compound Interest

An Example

- Suppose that you were going to invest \$5000 in an IRA earning interest at an annual rate of 5.5%
  - How would you determine the amount of interest you've made on your investment after one year?

$$i_1 = 5000(0.055) = \$275$$

An Example

- How much money would you have in your IRA account?

$$F_1 = 5000 + i_1 = 5000 + 5000(0.055) = 5000\left(1 + 0.055\right) = \$5275$$

- How much interest would you get after two years?

$$i_2 = 5275\left(0.055\right) = 290.13$$

An Example

- How much money would you have in your IRA account after two years?

$$F_2 = 5275 + i_2 = 5275 + 5275(0.055) = 5275(1 + 0.055) = 5000(1 + 0.055)(1 + 0.055)$$

- What about 10 years?

$$F_{10} = 5000(1 + 0.055)^{10} \approx \$8540.72$$

Compound Interest

- Notice that the interest in our account was paid at regular intervals, in this case every year, while our money remained in the account. This is called *compounding annually or one time per year*.

Compound Interest

- Suppose that instead of collecting interest at the end of each year, we decided to collect interest at the end of each quarter, so our interest is paid four times each year. What would happen to our investment?
- Since our account has an interest rate of 5.5% annually, we need to adjust this rate so that we get interest on a quarterly basis. The quarterly rate is:

$$5.5/4 = 1.375\%$$

Compound Interest

- So for our IRA account of $5000 at the end of a year looks like:

$$F_1 = 5000\left(1 + \frac{0.055}{4}\right)^{4\cdot 1} \approx \$5280.72$$

- After 10 years, we have:

$$F_{10} = 5000\left(1 + \frac{0.055}{4}\right)^{4\cdot 10} \approx \$8633.85$$

Compound Interest Formula

- *P* dollars invested at an annual rate *r*, compounded *n* times per year, has a value of *F* dollars after *t* years.

$$F = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}$$

- Think of *P* as the present value, and *F* as the future value of the deposit.

Effective Interest

- One may compare different interest rates and different frequencies of compounding by looking at the value of $1 at the end of one year.
- Such a rate is called the effective annual yield, annual percentage yield, or simply *effective interest*.

Effective Interest

- In the previous example, compounding quarterly, after one year we had:

$$F_1 = 5000\left(1+\frac{0.055}{4}\right)^{4\cdot 1} \approx \$5280.72$$

- We made \$280.72 in interest after 1 year compounded quarterly. That is a gain of $^{280.72}/_{5000}$ = 0.0561 = 5.61%.

Effective Interest

- To find the *effective interest* then, we find the amount made on \$1 for 1 year:

$$y = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{0.055}{4}\right)^4 - 1 = 0.0561$$

More on Effective Interest

- If we need the *annual rate* that would give an effective rate at a specified compounding, we need to solve for *r* :

$$y = \left(1 + \frac{r}{n}\right)^n - 1$$

$$y + 1 = \left(1 + \frac{r}{n}\right)^n$$

$$(y+1)^{1/n} = 1 + \frac{r}{n}$$

$$(y+1)^{1/n} - 1 = \frac{r}{n}$$

$$n\left[(y+1)^{1/n} - 1\right] = r$$

Compound Interest Formula

- Notice that when we collect interest more times each year, i.e. compounding more frequently, the amount of money is greater than only collecting interest once a year.