Composite Functions

What Are Composite Functions?

Composition of functions is when one function is inside of another function. For example, if we look at the function $h(x) = (2x - 1)^2$. We can say that this function, h(x), was formed by the composition of two other functions, the inside function and the outside function. In the case of $h(x) = (2x - 1)^2$, the inside function is $2x - 1$ and the outside function is z^2, the letter z was used just to represent a different variable, we could have used any letter that we wanted. Notice that if we put the inside function, $2x - 1$, into the outside function, z^2, we would get $z^2 = (2x - 1)^2$, which is our original function h(x).

The notation used for the composition of functions looks like this, $(f \circ g)(x)$. So what does this mean $(f \circ g)(x)$, the composition of the function f with g is defined as follows:

$(f \circ g)(x) = f(g(x))$, notice that in the case the function g is inside of the function f.

In composite functions it is very important that we pay close attention to the order in which the composition of the functions is written. In many cases $(f \circ g)(x)$ is not the same as $(g \circ f)(x)$. Let's look at why the order is so important:

$(f \circ g)(x) = f(g(x))$, the g function is inside of the f function

$(g \circ f)(x) = g(f(x))$, the f function is inside of the g function

$(f \circ g)(x)$ and $(g \circ f)(x)$ are often different because in the composite $(f \circ g)(x)$, f(x) is the outside function and g(x) is the inside function. Whereas in the composite $(g \circ f)(x)$, g(x) is the outside function and f(x) is the inside function. This difference in order will often be the reason why we will get different answers for $(f \circ g)(x)$ and $(g \circ f)(x)$. This means we need to make sure that we pay close attention to the way the problem is written when we are trying to find the composition of two functions.

How Do You Find the Composition of Two Functions?

Here are the steps we can use to find the composition of two functions:

Step 1: Rewrite the composition in a different form. For example, the composition $(f \circ g)(x)$ needs to rewritten as $f(g(x))$.

Step 2: Replace each occurrence of x found in the outside function with the inside function. For example, in the composition of $(f \circ g)(x) = f(g(x))$, we need to replace each x found in f(x), the outside function, with g(x), the inside function.

Step 3: Simplify the answer.

Examples – Now let's use the steps shown above to work through some examples.

Example 1: If $f(x) = -4x + 9$ and $g(x) = 2x - 7$, find $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x))$	Rewrite the composition in a different form.
$= -4(2x - 7) + 9$	Replace each occurrence of x in f(x) with $g(x) = 2x - 7$.
$= -8x + 28 + 7$ $= -8x + 37$	Simplify the answer by distributing and combining like terms.

Thus, $(f \circ g)(x) = -8x + 37$.

Example 2: If $f(x) = -4x + 9$ and $g(x) = 2x - 7$, find $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x))$	Rewrite the composition in a different form.
$= 2(-4x + 9) - 7$	Replace each occurrence of x in g(x) with $f(x) = -4x + 9$.
$= -8x + 18 - 7$ $= -8x + 11$	Simplify the answer by distributing and combining like terms.

Thus, $(g \circ f)(x) = -8x + 11$.

Notice that in Examples 1 and 2 the functions $f(x) = -4x + 9$ and $g(x) = 2x - 7$ were the same, but $(f \circ g)(x)$ and $(g \circ f)(x)$ produced different answers. These two examples should help us understand why we need to be very specific when we are asked to find either $(f \circ g)(x)$ or $(g \circ f)(x)$. The way we write down the problem can make a big difference in our answer.

Example 3: If $h(x) = 3x - 5$ and $g(x) = 2x^2 - 7x$, find $(g \circ h)(x)$.

$(g \circ h)(x) = g(h(x))$	Rewrite the composition in a different form.
$= 2(3x - 5)^2 - 7(3x - 5)$	Replace each occurrence of x in g(x) with $h(x) = 3x - 5$.
$= 2(9x^2 - 30x + 25) - 7(3x - 5)$ $= 18x^2 - 60x + 50 - 21x + 35$ $= 18x^2 - 81x + 85$	Simplify the answer by first dealing with the exponent and squaring $(3x - 5)$, then distributing, and finally combining like terms.

Thus, $(g \circ h)(x) = 18x^2 - 81x + 85$.

Addition Examples

If you would like to see more examples of composition of functions, just click on the link below.

Additional Examples

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: If $f(x) = x^2 - 4x + 2$ and $g(x) = 3x - 7$, find $(f \circ g)(x)$.

Problem 2: If $g(x) = -6x + 5$ and $h(x) = -9x - 11$, find $(g \circ h)(x)$.

Problem 3: If $f(x) = \sqrt{2x - 5}$ and $g(x) = 5x^2 - 3$, find $(g \circ f)(x)$.

Problem 4: If $f(x) = -2x + 9$ and $g(x) = -4x^2 + 5x - 3$, find $(f \circ g)(x)$.

Problem 5: If $f(x) = x - 3$ and $g(x) = 4x^2 - 3x - 9$, find $(g \circ f)(x)$.

Problem 6: If $g(x) = \sqrt[3]{x - 4}$ and $h(x) = x^3 + 4$, find $(h \circ g)(x)$.

Solutions to Practice Problems