Combining and Describing Functions

Functions

1. Inverse Functions

You already know that exponential functions and logarithmic functions are inverses of one another. Now, let's see what is implied by the term "inverse".

Given $\boldsymbol{f(x) = 2e^{3x} + 1}$

(a) Find the inverse, $f^{-1}(x)$.

(b) Now fill in the table below for $f(x)$ and $f^{-1}(x)$.

$\boldsymbol{x}$	**0**	**1**	**2**	**3**	**4**
$\boldsymbol{f(x)}$					
$\boldsymbol{f^{-1}(x)}$					

Is there a specific pair of points that stand out to you?

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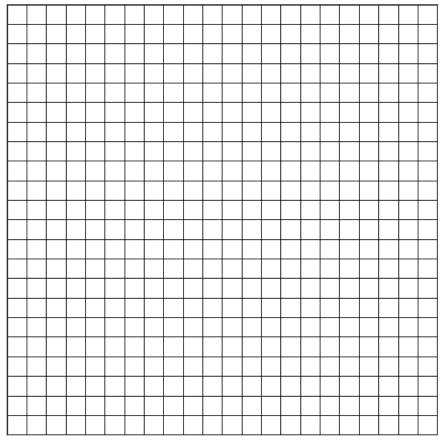
(c) Sketch the graph of both $f(x)$ and $f^{-1}(x)$ on the same axes below.



(d) How do the graphs compare to one another?

(e) For $f(x)$, give the domain, range, and the equation (and type) of the asymptote.

D: R: A:

(f) For $f^{-1}(x)$, give the domain, range, and the equation (and type) of the asymptote.

D: R: A:

Now, let's look at an anonymous function represented by a table of values.

$\boldsymbol{x}$	0	1	2	3	4
$\boldsymbol{f(x)}$	0	1	1	5	3

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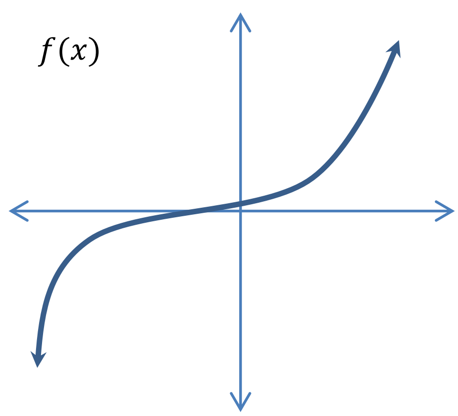
(a) Using the table below, give a table of values for the inverse of the above function.

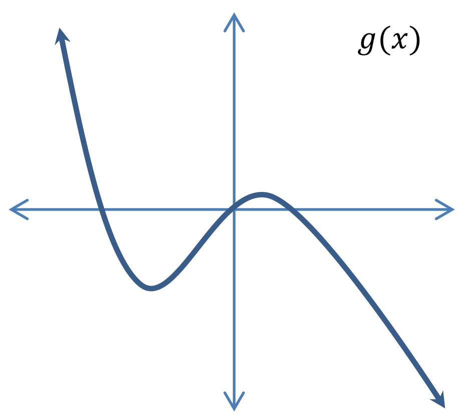
x					
$f^{-1}(x)$					

(b) Is the inverse a function? How can you tell?

One-to-one functions are functions that have an inverse that is also a function. You can tell graphically if a function is one-to-one without graphing the inverse – it must pass the horizontal line test.

Consider the following functions:





(a) Is either function one-to-one?

(b) Draw the inverse function on each graph.

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Find the inverse of the following and give their domain.

1. $g(x) = \frac{3}{x-1}$

2. $f(x) = x^2 - 1$

3. $f(x) = \sqrt[3]{\frac{x-7}{3}}$

4. $h(x) = log_3(x^2 + 2)$

We'll revisit inverse functions in a moment.

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2. Combining and Compositions

I. Basic Combinations and Compositions

For 1 – 8 below, use the following information. Give the domain of each.

$\boldsymbol{f(x) = -2x^2 - 2x + 1}$ and $\boldsymbol{g(x) = x + 1}$

1. Find $f(x) + g(x)$

2. Find $f(x) - g(x)$

3. Find $f(x) \cdot g(x)$

4. Find $\frac{f(x)}{g(x)}$

5. Find $f(g(x))$

6. Find $g(f(x))$

7. Find $g(g(x))$

8. Find $f(g(-1))$

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II. Function Compositions Using Sets of Points

f={(–2, 3), (–1, 1), (0, 0), (1, –1), (2, –3)}

g={(–3, 1), (–1, –2), (0, 2), (2, 2), (3, 1)}

Using the information above, find the following:

1. f(1)
2. g(-1)
3. g(f(1))
4. f(g(0))
5. f(g(-1))
6. g(f(-1))

Given two functions, $f(x)$ and $g(x)$, evaluate the following given that:

For $f(x)$: $f(-2) = 5,\ f(-1) = 2,\ f(0) = -1,\ f(1) = -3,\ f(2) = 3$

For $g(x)$: $g(-2) = -1,\ g(-1) = -2,\ g(0) = 0,\ g(1) = 2,\ g(2) = 3$

1. $(f + f)(0)$
2. $(f - g)(-2)$
3. $f(g(-1))$
4. $g(f(0))$
5. $f\big(g(1)\big) - g(f(-1))$
6. $f^{-1}(f(2))$

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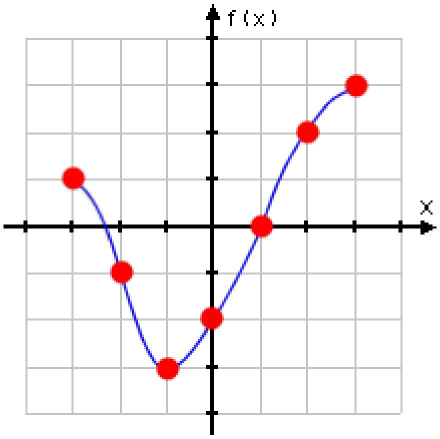
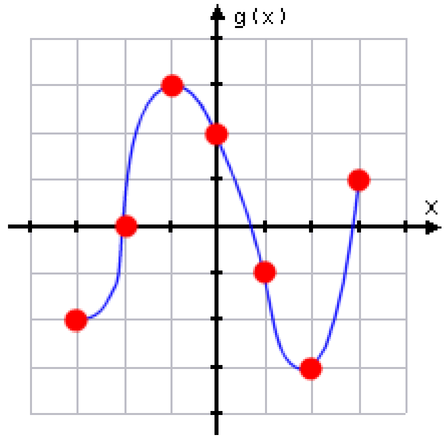
III. Function Compositions Using Graphs





Given $f(x)$ and $g(x)$ as shown in the graphs above, find the following:

1. $f(g(1))$

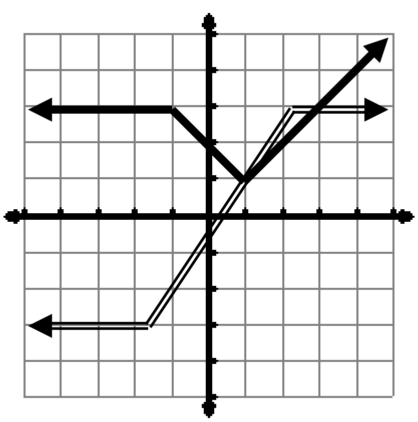
2. $g(f(-2))$

3. $f(f(0))$

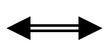
4. $f^{-1}(g(2))$

5. $g(f^{-1}(2))$

6. $g^{-1}(g^{-1}(1))$



$f(x) =$ 

$g(x) =$ 

Use the graph to the left for the following:

1. $(f+f)(2)$

2. $f(g(1))$

3. $g(f(-1))$

4. $f^{-1}(f^{-1}(1))$

5. $(f+g)(3)$

6. $f(4) - g(-1)$

7. $(f-g)(-3) + f(f(2))$

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IV. Composition Extensions and Applications

1. Show that $f(x) = 2x^2 - 1$ and $g(x) = \sqrt{\frac{x+1}{2}}$ are inverse functions using compositions.

2. Verify that $f(x) = \sqrt{\frac{x-2}{3}}$ and $g(x) = 3x^2 + 2$ are inverses.

3. Given $f(x) = \sqrt{x}$ and $g(x) = x - 2$, find the domains of $f(g(x))$ and $g(f(x))$.

4. Given $h(x) = (x+1)^2 + 2(x+1) - 3$, determine two functions $f(x)$ and $g(x)$ which, when composed, generate $h(x)$.

5. Given $h(x) = \sqrt{4x+1}$, determine two functions $f(x)$ and $g(x)$ which, when composed, generate $h(x)$.

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6. Given $h(x) = \frac{(3x-1)^2}{5}$, determine two functions $f(x)$ and $g(x)$ which, when composed, generate $h(x)$.

7. You work forty hours a week at a furniture store. You receive a $220 weekly salary, plus a 3% commission on sales over $5000. Assume that you sell enough this week to get the commission. Given the functions $f(x) = 0.03x$ and $g(x) = x - 5000$, which composed function, $f(g(x))$ or $g(f(x))$, represents your commission?

8. You make a purchase at a local hardware store, but what you've bought is too big to take home in your car. For a small fee, you arrange to have the hardware store deliver your purchase for you. You pay for your purchase, plus the sales taxes, plus the fee. The taxes are 7.5% and the fee is $20.

(a) Write a function $t(x)$ for the total, after taxes, on purchase amount x. Write another function $f(x)$ for the total, including the delivery fee, on purchase amount x.

(b) Calculate and interpret $f(t(x))$ and $t(f(x))$. Which results in a lower cost to you?

(c) Suppose taxes, by law, are not to be charged on delivery fees. Which composite function must then be used?

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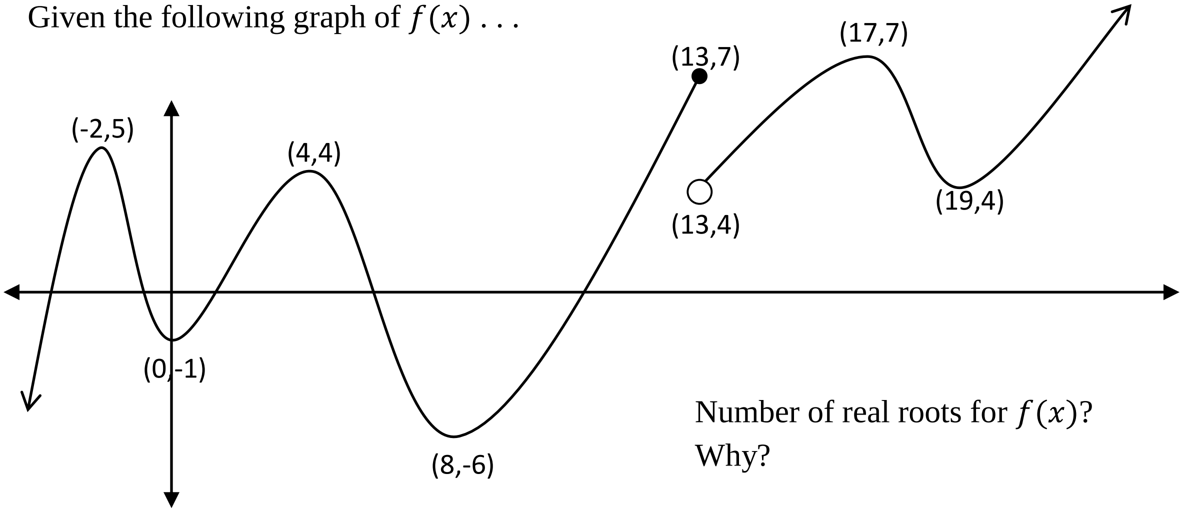
3. Describing Functions

Given the following graph of $f(x)$ . . .



Number of real roots for $f(x)$?
Why?

Complete the table below.

x	f(x)	f(x)+2	f(x)-1	-f(x)	2f(x)	$-\frac{1}{2}$f(x)	$\lvert f(x) \rvert$
-2							
0							
4							
8							
13							
17							
19							

- How can you tell that $f(x)$ is a function? Explain.
- What is the domain and range of $f(x)$?
- Is $f(x)$ a continuous function? How can you tell?
- What is the end behavior of $f(x)$?
- Give the intervals of increase and decrease and local maximums and minimums for $f(x)$.
- What is $f(2x)$ if $x = 4$?