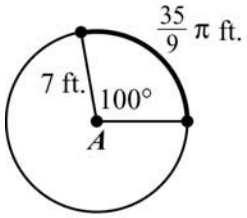
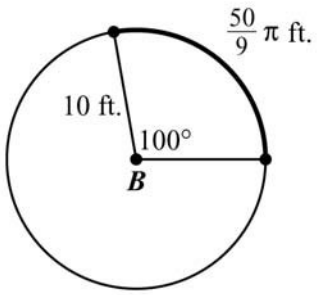
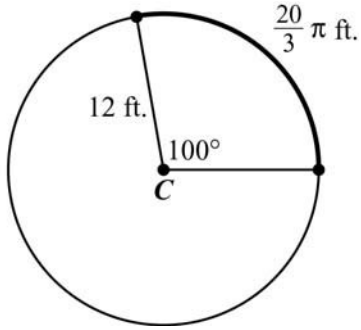
REVIEW EXAMPLES

1. Circles *A*, *B*, and *C* have a central angle measuring 100°. The length of each radius and the length of each intercepted arc are shown.







   a. What is the ratio of the radius of circle *B* to the radius of circle *A*?
   b. What is the ratio of the length of the intercepted arc of circle *B* to the length of the intercepted arc of circle *A*?
   c. Compare the ratios in parts (a) and (b).
   d. What is the ratio of the radius of circle *C* to the radius of circle *B*?
   e. What is the ratio of the length of the intercepted arc of circle *C* to the length of the intercepted arc of circle *B*?
   f. Compare the ratios in parts (d) and (e).
   g. Based on your observations of circles *A*, *B*, and *C*, what conjecture can you make about the length of the arc intercepted by a central angle and the radius?
   h. What is the ratio of arc length to radius for each circle?

Solution:

a. Divide the radius of circle *B* by the radius of circle *A*: $\frac{\text{circle B}}{\text{circle A}} = \frac{10}{7}$

b. Divide the length of the intercepted arc of circle *B* by the length of the intercepted arc of circle *A*:

$$\frac{\frac{50}{9}\pi}{\frac{35}{9}\pi} = \frac{50\pi}{9} \cdot \frac{9}{35\pi} = \frac{10}{7}$$

c. The ratios are the same.

d. Divide the radius of circle *C* by the radius of circle *B*: $\frac{\text{circle C}}{\text{circle B}} = \frac{12}{10} = \frac{6}{5}$

e. Divide the length of the intercepted arc of circle *C* by the length of the intercepted arc of circle *B*: $\frac{\frac{20}{3}\pi}{\frac{50}{9}\pi} = \frac{20\pi}{3} \cdot \frac{9}{50\pi} = \frac{6}{5}$

f. The ratios are the same.

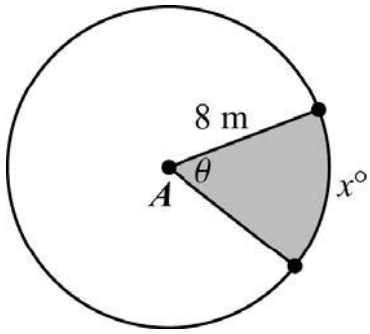
g. When circles, such as circles *A*, *B*, and *C*, have the same central angle measure, the ratio of the lengths of the intercepted arcs is the same as the ratio of the radii.

h. Circle *A*: $\frac{\frac{35}{9}\pi}{7} = \frac{35}{63}\pi = \frac{5}{9}\pi$

Circle *B*: $\frac{\frac{50}{9}\pi}{10} = \frac{50}{90}\pi = \frac{5}{9}\pi$

Circle *C*: $\frac{\frac{20}{3}\pi}{12} = \frac{20}{36}\pi = \frac{5}{9}\pi$

2. Circle *A* is shown.



If $x = 50$, what is the area of the shaded sector of circle *A*?

Solution:

To find the area of the sector, divide the measure of the central angle of the arc in degrees by 360, and then multiply that amount by the area of the circle. The arc measure, *x*, is equal to the measure of the central angle, θ. The formula for the area of a circle is $A = \pi r^2$.

$A_{sector} = \frac{\pi r^2 \theta}{360}$ Area of sector of a circle with radius *r* and central angle θ in degrees

$A_{sector} = \frac{50\pi(8)^2}{360}$ Substitute 50 for θ and 8 for *r*.

$A_{sector} = \frac{5\pi(64)}{36}$ Rewrite the fraction and the power.

$A_{sector} = \frac{320\pi}{36}$ Multiply.

$A_{sector} = \frac{80\pi}{9}$ Rewrite.

The area of the sector is $\frac{80}{9}\pi$ square meters.