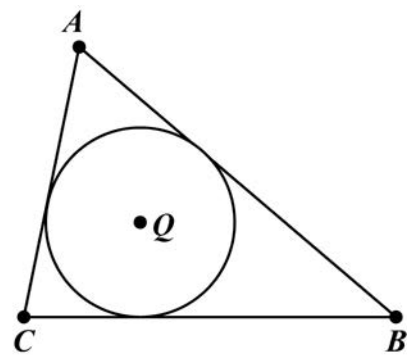
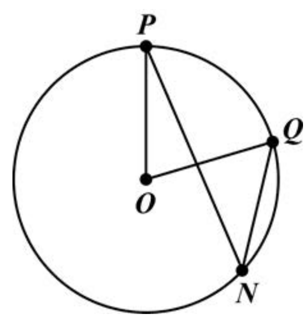
24. An ***inscribed circle*** is a circle enclosed in a polygon, where every side of the polygon is tangent to the circle. Specifically, when a circle is inscribed in a triangle, the center of the circle is the ***incenter*** of the triangle. The incenter is equidistant from the sides of the triangle. Circle *Q* is inscribed in triangle *ABC*, and point *Q* is the incenter of the triangle. Notice also that the sides of the triangle form circumscribed angles with the circle.



REVIEW EXAMPLES

1. $\angle PNQ$ is inscribed in circle *O* and $m\overset{\frown}{PQ} = 70°$.



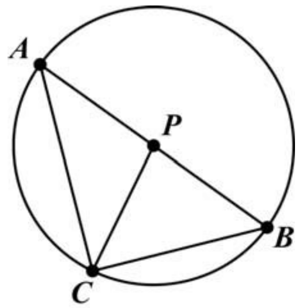
a. What is the measure of $\angle POQ$?
b. What is the relationship between $\angle POQ$ and $\angle PNQ$?
c. What is the measure of $\angle PNQ$?

Solution:

a. The measure of a central angle is equal to the measure of the intercepted arc. $m\angle POQ = m\overset{\frown}{PQ} = 70°$.
b. $\angle POQ$ is a central angle that intercepts $\overset{\frown}{PQ}$. $\angle PNQ$ is an inscribed angle that intercepts $\overset{\frown}{PQ}$. The measure of the central angle is equal to the measure of the intercepted arc. The measure of the inscribed angle is equal to one-half the measure of the intercepted arc. So $m\angle POQ = m\overset{\frown}{PQ}$ and $m\angle PNQ = \frac{1}{2}m\overset{\frown}{PQ}$, so $m\angle POQ = 2m\angle PNQ$.
c. From part (b), $m\angle POQ = 2m\angle PNQ$

Substitute: $70° = 2m\angle PNQ$
Divide: $35° = m\angle PNQ$

2. In circle P below, $\overline{AB}$ is a diameter.



If $m\angle APC = 100°$, find the following:

a. $m\angle BPC$
b. $m\angle BAC$
c. $m\overset{\frown}{BC}$
d. $m\overset{\frown}{AC}$

Solution:

a. $\angle APC$ and $\angle BPC$ are supplementary, so $m\angle BPC = 180° - m\angle APC$, so $m\angle BPC = 180° - 100° = 80°$.

b. $\angle BAC$ is an angle in $\triangle APC$. The sum of the measures of the angles of a triangle is 180°.

For $\triangle APC$: $m\angle APC + m\angle BAC + m\angle ACP = 180°$

You are given that $m\angle APC = 100°$.

Substitute: $100° + m\angle BAC + m\angle ACP = 180°$

Subtract 100° from both sides: $m\angle BAC + m\angle ACP = 80°$

Because two sides of $\triangle APC$ are radii of the circle, $\triangle APC$ is an isosceles triangle. This means that the two base angles are congruent, so $m\angle BAC = m\angle ACP$.

Substitute: $m\angle BAC$ for $m\angle ACP$: $m\angle BAC + m\angle BAC = 80°$

Add: $2m\angle BAC = 80°$

Divide: $m\angle BAC = 40°$

You could also use the answer from part (a) to solve for $m\angle BAC$. Part (a) shows $m\angle BPC = 80°$.

Because the central angle measure is equal to the measure of the intercepted arc, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

Because an inscribed angle is equal to one-half the measure of the intercepted arc, $m\angle BAC = \frac{1}{2}m\overset{\frown}{BC}$.

By substitution: $m\angle BAC = \frac{1}{2}(80°)$

Therefore, $m\angle BAC = 40°$.

c. $\angle BAC$ is an inscribed angle intercepting $\overset{\frown}{BC}$. The intercepted arc is twice the measure of the inscribed angle.

$m\overset{\frown}{BC} = 2m\angle BAC$

From part (b), $m\angle BAC = 40°$.

Substitute: $m\overset{\frown}{BC} = 2 \cdot 40°$

$m\overset{\frown}{BC} = 80°$

You could also use the answer from part (a) to solve. Part (a) shows $m\angle BPC = 80°$. Because $\angle BPC$ is a central angle that intercepts $\overset{\frown}{BC}$, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

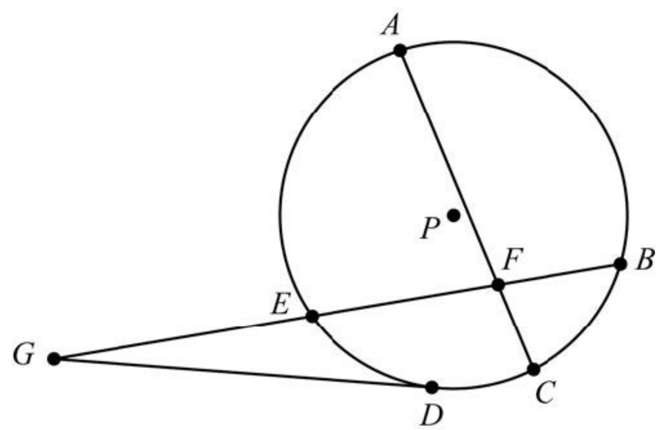
d. $\angle APC$ is a central angle intercepting $\overset{\frown}{AC}$. The measure of the intercepted arc is equal to the measure of the central angle.

$m\overset{\frown}{AC} = m\angle APC$

You are given $m\angle APC = 100°$.

Substitute: $m\overset{\frown}{AC} = 100°$

3. In circle P below, $\overline{DG}$ is a tangent. $AF = 8$, $EF = 6$, $BF = 4$, and $EG = 8$.



Find CF and DG.

Solution:

First, find CF. Use the fact that $\overline{CF}$ is part of a pair of intersecting chords.

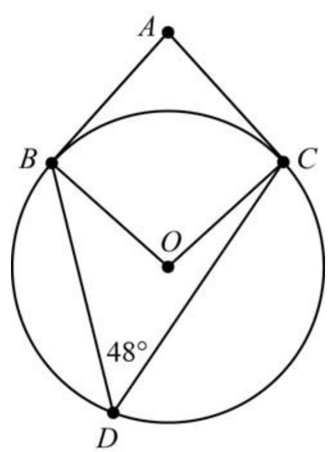
$$AF \cdot CF = EF \cdot BF$$
$$8 \cdot CF = 6 \cdot 4$$
$$8 \cdot CF = 24$$
$$CF = 3$$

Next, find DG. Use the fact that $\overline{DG}$ is tangent to the circle.

$$EG \cdot BG = DG^2$$
$$8 \cdot (8+6+4) = DG^2$$
$$8 \cdot 18 = DG^2$$
$$144 = DG^2$$
$$\pm 12 = DG$$
$$12 = DG \text{ (since length cannot be negative)}$$

$CF = 3$ and $DG = 12$.

4. In this circle, $\overline{AB}$ is tangent to the circle at point B, $\overline{AC}$ is tangent to the circle at point C, and point D lies on the circle. What is $m\angle BAC$?



Solution:

Method 1

First, find the measure of angle BOC. Angle BDC is an inscribed angle, and angle BOC is a central angle.

$m\angle BOC = 2 \cdot m\angle BDC$

$= 2 \cdot 48°$

$= 96°$

Angle BAC is a circumscribed angle. Use the measure of angle BOC to find the measure of angle BAC.

$m\angle BAC = 180° - m\angle BOC$

$= 180° - 96°$

$= 84°$

Method 2

Angle *BDC* is an inscribed angle. First, find the measures of $\overset{\frown}{BC}$ and $\overset{\frown}{BDC}$.

$$m\angle BDC = \frac{1}{2} \bullet m\overset{\frown}{BC}$$

$$48^\circ = \frac{1}{2} \bullet m\overset{\frown}{BC}$$

$$2 \bullet 48^\circ = m\overset{\frown}{BC}$$

$$96^\circ = m\overset{\frown}{BC}$$

$$\begin{aligned} m\overset{\frown}{BDC} &= 360^\circ - m\overset{\frown}{BC} \\ &= 360^\circ - 96^\circ \\ &= 264^\circ \end{aligned}$$

Angle *BAC* is a circumscribed angle. Use the measures of $\overset{\frown}{BC}$ and $\overset{\frown}{BDC}$ to find the measure of angle *BAC*.

$$\begin{aligned} m\angle BAC &= \frac{1}{2}\left(m\overset{\frown}{BDC} - m\overset{\frown}{BC}\right) \\ &= \frac{1}{2}\left(264^\circ - 96^\circ\right) \\ &= \frac{1}{2}\left(168^\circ\right) \\ &= 84^\circ \end{aligned}$$