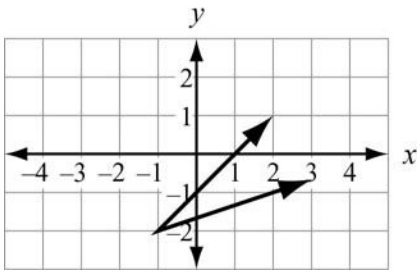
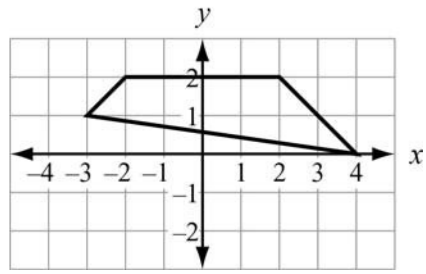
REVIEW EXAMPLES

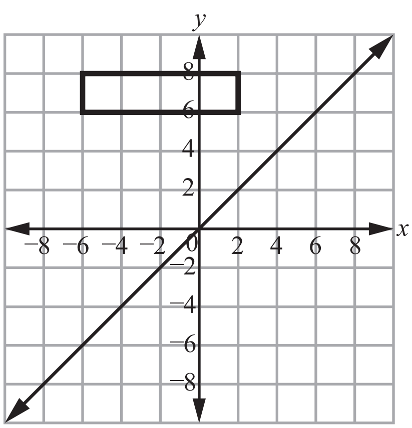
1. Draw the image of each figure, using the given transformation.

a. 

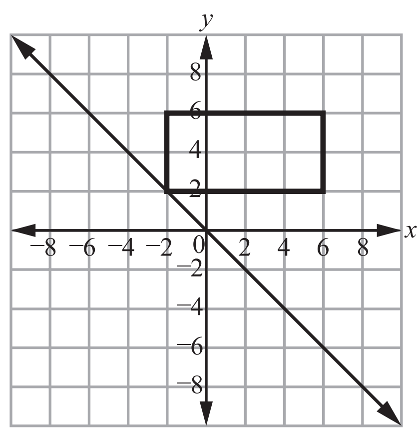
Use the translation $(x, y) \rightarrow (x - 3, y + 1)$.

b. 

Reflect across the x-axis.

c. 

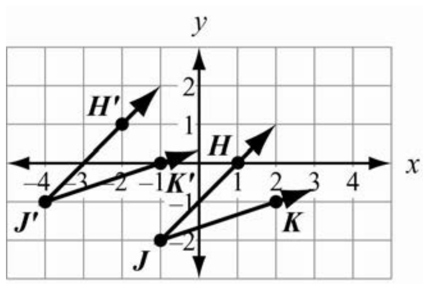
Reflect across the line $y = x$.

d. 

Identify the vertices. The reflection image of each point (x, y) across the line $y = -x$ is $(-y, -x)$.

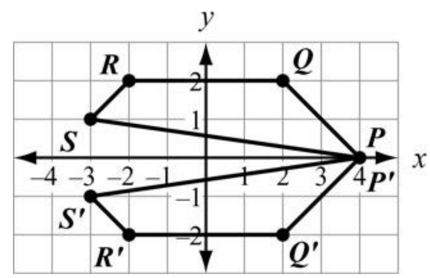
Solution:

a.



Identify the vertex and a point on each side of the angle. Translate each point 3 units left and 1 unit up. The image of given ∠*HJK* is ∠*H*′*J*′*K*′.

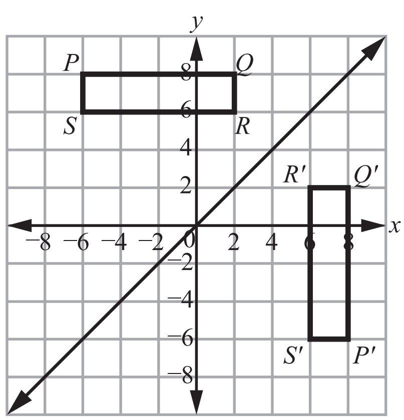
b.



Identify the vertices. The reflection image of each point (*x*, *y*) across the *x*-axis is (*x*, –*y*).

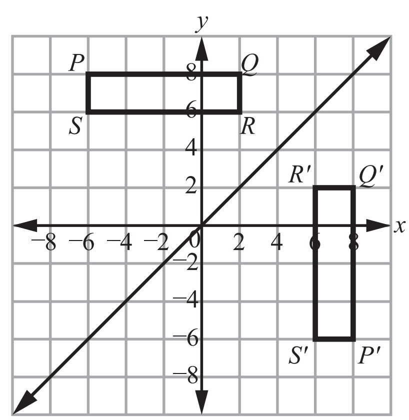
The image of given polygon *PQRS* is *P*′*Q*′*R*′*S*′, where *P* and *P*′ are the same.

c.



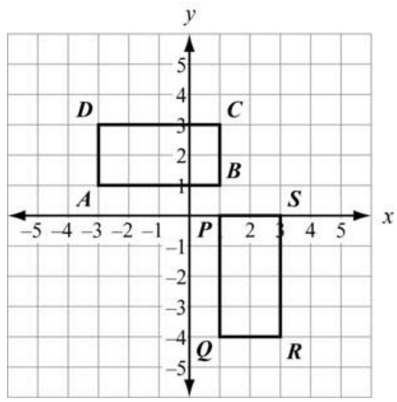
Identify the vertices. The reflection image of each point (*x*, *y*) across the line *y* = *x* is (*y*, *x*).

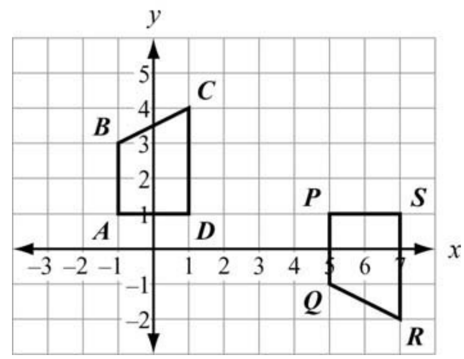
d.



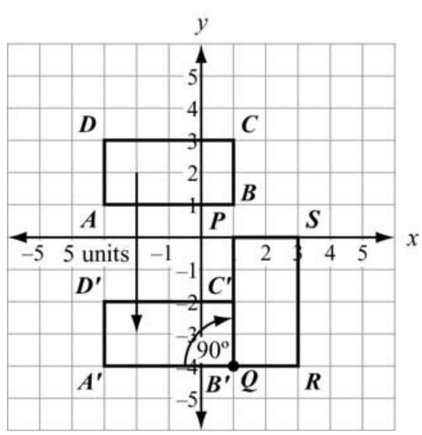
Identify the vertices. The reflection image of each point (*x*, *y*) across the line *y* = –*x* is (–*y*, –*x*).

2. Specify a sequence of transformations that will map *ABCD* to *PQRS* in each case.

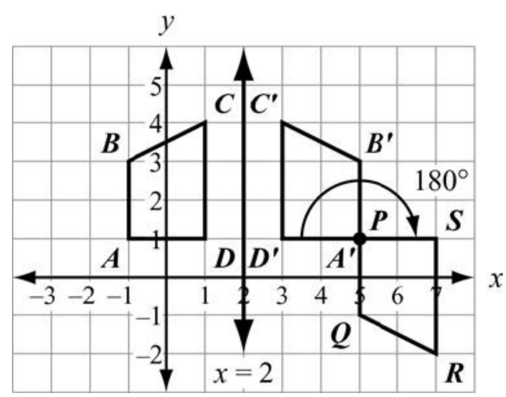
a. 

b. 

Solution:

a. 

Translate *ABCD* down 5 units to obtain *A′B′C′D′*. Then rotate *A′B′C′D′* clockwise 90° about point *B′* to obtain *PQRS*.

b. 

Reflect *ABCD* across the line $x = 2$ to obtain *A′B′C′D′*. Then rotate *A′B′C′D′* 180° about point *A′* to obtain *PQRS.*

Note that *A′* and *P* are the same point.

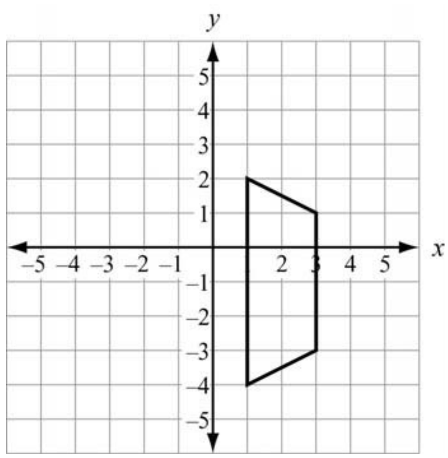
Note that there are other sequences of transformations that will also work for each case.

Important Tip

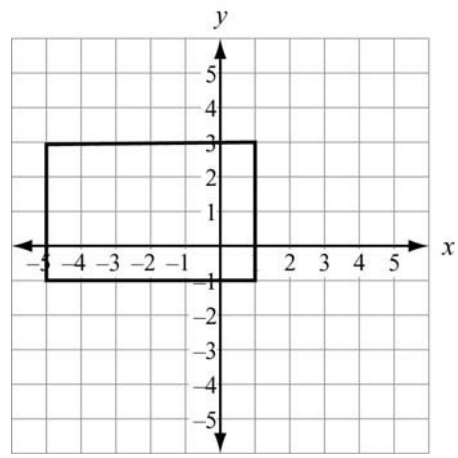
✍ A 180° rotation clockwise is equivalent to a 180° rotation counterclockwise.

3. Describe every transformation that maps each given figure to itself.

a.

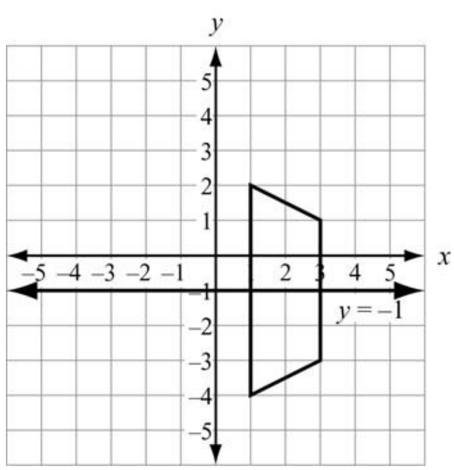


b.



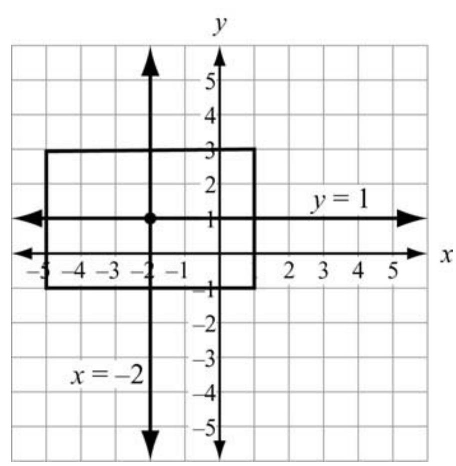
Solution:

a.



There is only one transformation: Reflect the figure across the line $y = -1$.

b.



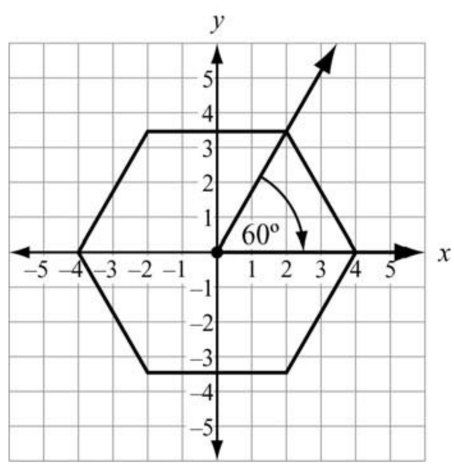
There are three transformations:

- reflect across the line $y = 1$, or
- reflect across the line $x = -2$, or
- rotate 180° about the point (–2, 1).

4. Describe every transformation that maps this figure to itself: a regular hexagon (6 sides) that is centered about the origin and that has a vertex at (4, 0).

Solution:

The angle formed by any two consecutive vertices and the center of the hexagon measures 60° because $\frac{360°}{6} = 60°$. So a rotation about the origin, clockwise or counterclockwise, of 60°, 120°, or any other multiple of 60° maps the hexagon to itself.



If a reflection across a line maps a figure to itself, then that line is called a ***line of symmetry***.

A regular hexagon has 6 lines of symmetry: 3 lines through opposite vertices and 3 lines through midpoints of opposite sides.

A reflection across any of the 6 lines of symmetry maps the hexagon to itself.